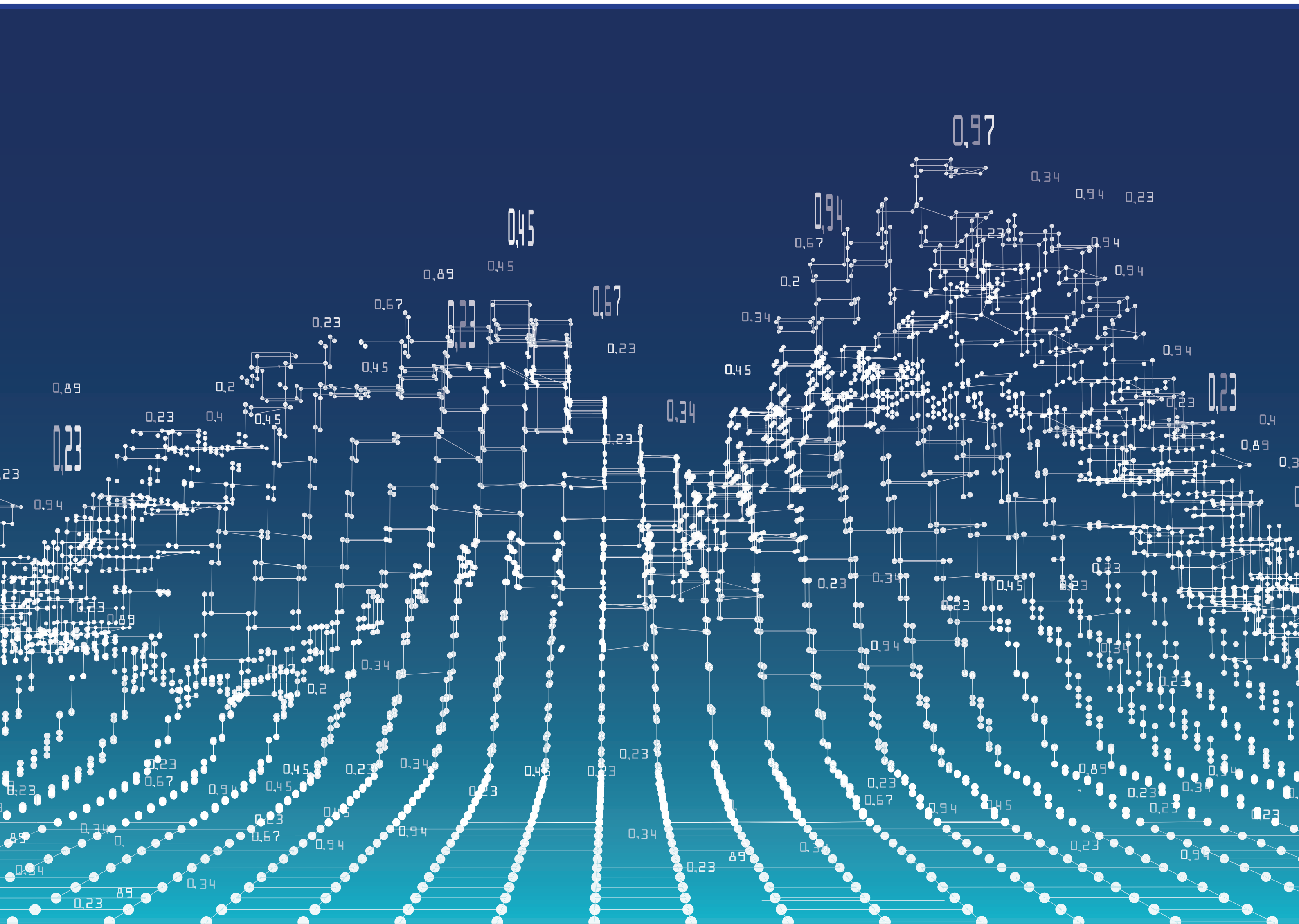
RIGA TECHNICAL UNIVERSITY

Jegors Fjodorovs

RISK FORECAST WITH CONTINUOUS MODELS FOR EVALUATING TECHNOLOGY AND MARKETS

Summary of the Doctoral Thesis



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RIGA TECHNICAL UNIVERSITY
Faculty of Computer Science and Information Technology
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Summary of the Doctoral Thesis

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DOCTORAL THESIS PROPOSED TO RIGA TECHNICAL UNIVERSITY FOR THE PROMOTION TO THE SCIENTIFIC DEGREE OF DOCTOR OF ENGINEERING SCIENCES

To be granted the scientific degree of Doctor of Engineering Sciences, the present Doctoral Thesis has been submitted for the defence at the open meeting of RTU Promotion Council on June 3, 2019 at the Faculty of Computer Science and Information Technology of Riga Technical University, 1 Sētas Street, Room 202.

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DECLARATION OF ACADEMIC INTEGRITY

I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Engineering Sciences is my own. I confirm that this Doctoral Thesis had not been submitted to any other university for the promotion to a scientific degree.

Jegors Fjodorovs ……………………………. (signature)
Date: ………………………

The Doctoral Thesis has been written in Latvian. It consists of Introduction; four sections; Conclusions; 5 appendixes; 40 figures; 7 tables; the total number of pages is 134. The Bibliography contains 94 titles.

CONTENTS

GENERAL OVERVIEW OF THE THESIS

Motivation

Information theory understands the transmission, processing, extraction and use of information. In this way, information can be pre received as uncertainty resolution [1]. Nowadays, private and public companies all around the world collect different types of information and use stochastic modeling in forecasting with the purpose to estimate the expected (forecasted) interest rate term structure, exchange rates, inflation rates, electricity demand in different time periods, prices of derivative instruments, etc., to implement an optimal state budget management policy, limit financial risks, model inventory levels and prices of different financial assets. The flow of the abovementioned information [2] is continuously increasing, and modern forecasting developers face big data, the processing techniques of which may hinder decision-making. Therefore, fast information processing algorithms based on nonparametric models are needed. In this regard, the promotion work has created iterative forecasting procedures based on cumulative regressions.

It is essential to forecast future values of financial assets, technical parameters and macroeconomic data. One of the most common forecasting tools is the Box–Jenkins (Box and Jenkins, 1970) autoregressive integrated moving average (ARIMA) model. However, the Box–Jenkins methodology has its own shortcomings (it does not account for thick tails and high peaks (leptokurtic) of asset return distribution). Nowadays forecasting in fields like finance is related to work with heteroscedastic data (volatility is not stationary – spikes of high volatility appear in different time periods). Therefore, a responsible practitioner has to use forecasting methods that are able to react to changes in volatility and serial correlations. Heston [71] was one of the first to deal with stochastic oscillation models and suggested describing the dispersion process with a separate stochastic, but did not focus on verifying the model's compliance with real data. The importance of the topic is underscored by the fact that there exists plentiful software that calculate forecasting error, usually with the maximum likelihood method, but none of them tests whether stability conditions hold true for the error distribution [3]. In this way, the existing demand for forecasting methods, which are suitable for working with serial correlated data, makes this paper highly topical.

Research purpose and tasks

The purpose of the promotion work is to develop methods and algorithms for constructing forecasting models accounting for the non-linear dependency of observational errors. The work solves practical problems that appear in financial analysis, such as, how to find the true value of derivative instruments including call and put options using the abovementioned model, or how to construct a copula-type function in order to describe the impact of different financial risks on the price of a derivative instrument, as well as estimate the range of asset volatility in the case of a sudden huge uncertainty in financial markets.

To reach the research purpose, the following tasks have been formulated:

- to develop a method of constructing autoregressive model based on copula-type relations;
- to develop a method for incorporating the balance correlation into a stochastic differential equation and a transition to continuous time – to study the stationary of this equation;
- to introduce a correction for correlation into D. Nelson's conditional variance risk management model in the discrete time;
- to analyse the impact of observation errors' correlation on the Black-Scholes model parameters and recalculate the hedging factors.

Research object and subject

The object of the research is the time series. The main research subject is the conditional variance of observed errors.

Research methods

The research method is based on the formulated problem (models for analyzing data with autocorrelated residuals). The following methods are proposed and constitute practical base for solving the formulated tasks: analysis of linear regression residuals with non-stationary variance; R. Engle's (the Nobel awardee) method; J. Carkov's diffusion approximation method of conditional variance GARCH models; a copula-based non-linear regression construction method; and the risk management and limitation method by R. Merton and M. Scholes (both Nobel awardees). To develop an imitation apparatus, which is a set of operations necessary for the statistical approbation of the proposed models, commonly known stochastic models with added asset return autocorrelations have been used. The proposed model for solving the formulated research problem is the two-factor affine model, where the first factor is asset price and the second is its volatility. For successful representation of the model's results, a discrete representation of the imitation models and copula construction have been performed. The obtained models have been applied for financial markets data using Matlab software. All in all, the promotion work makes use of methods of mathematical statistics and probability theory, optimization theory and imitation modeling methods.

Scientific novelty

The following applications constitute the work's product.

1. The created algorithm for the construction of an autoregressive forecasting model without assumptions about rational expectation and the explicit form of forecasting error formula.

2. Including residual correlation from discrete time models when transforming to the continuous time, which results in a more precise forecasting model and risk calculation.

Practical importance

The proposed method and algorithm to construct autoregressive models in the discrete time can be used for risk analysis and forecasting given the sufficient size of the stationary sample.

The results regarding introducing a correlation correction allow to estimate the time of reaching stationary and distribution for the constructed risk component more precisely.

The promotion work has the following practical results:

- a forecast for the VIX index of a stock based on the discrete representation of the Heston model has been created;
- nonparametric regression for the VIX index has been found, which allows building new forecasts;
- an algorithm for modeling of the forecasted operational deficiencies of power equipment, which relates to the possibility to predict equipment safety, has been created;
- considering the autocorrelation of returns, stock option agreement of Tesla Motors Inc. has been repriced.

Thesis to be defended

1. The created algorithm for the construction of a copula based regression allows to create an autoregressive forecasting model without assumptions about the distribution of rational mathematical expectations and forecasting errors, the so-called algorithm for constructing a complex regression, has been created.
2. Inclusion of residual correlation from discrete time models when transforming to the continuous time allows us to construct a more precise forecasting model and calculate the risk.

Approbation of the results

The research has been approbated by presenting the results in 14 different international scholarly conferences and seminars and by publishing 10 articles in international scholarly journals.

Scientific conferences and workshops

1. An. Matvejevs, J. Fjodorovs, O. Pavlenko "Testing Heston Model Consistency and Evaluation of Parameters Thought Reprezentation in Discrete Time" 10th

International Conference on Applied Mathematics – APLIMAT 2011 in Bratislava, Slovakia, February 3–5, 2011.

2. An. Matvejevs, J. Fjodorovs “Copula Based Semiparametric Regression Models” 16th International Conference on Mathematical Modelling and Analysis, May 26–29, 2011, Sigulda, Latvia.
3. An. Matvejevs, J. Fjodorovs “The Impact of Serial Correlation on Financial Active Pricing and Risk Hedging” 17th International Conference on Mathematical Modelling and Analysis, June 6–9, 2012, Tallinn, Estonia.
4. An. Matvejevs, J. Fjodorovs “Copula Based Semiparametric Regressive Models” 11th International Conference on Applied Mathematics – APLIMAT 2012 in Bratislava, Slovakia, February 7–9, 2012.
5. J. Fjodorovs “Modeling Stohastic Proceses Using Copula Approach” RTU International Conference, October, Riga, Latvia, 2011.
6. J. Fjodorovs “Copula Based GARCH(1,1) Models” 18th International Conference on Mathematical Modelling and Analysis, May 27–31, 2013, Tartu, Estonia.
7. An. Matvejevs, J. Fjodorovs “Pricing of Financial Actives with Serial Correlation in Returns” RTU 53rd International Scientific Conference, October 11–12, Riga, Latvia, 2012.
8. An. Matvejevs, J. Fjodorovs “Copula Based Semiparametric Regressive Models” 12th International Conference on Applied Mathematics – APLIMAT 2013 in Bratislava, Slovakia, February 5–7, 2013.
9. An. Matvejevs, J. Fjodorovs “Revaluation of Estimated Option Prices Using GARCH Processes with Most Preferable Properties” RTU 54th International Scientific Conference, October 14–16, 2013, Riga, Latvia.
10. An. Matvejevs, J. Fjodorovs “Revaluation of Estimated Option Prices Using GARCH Processes with Most Preferable Properties” 7th International Conference on Computational and Financial Econometrics (CFE 2013), December, 15–19, 2014, London, Great Britain.
11. J. Fjodorovs “Simulation of Option Prices Using GARCH Processes for Autocorrelated Stock Returns” 13th International Conference on Applied Mathematics – APLIMAT 2014 in Bratislava, February 4–6, 2014.
12. J. Fjodorovs “Copulas and Markov chains” 10th Latvian International Mathematical conference, April 11–12, 2014, Liepāja, Latvia.
13. J. Fjodorovs “Copula Fit to the Nonlinear Processes in the Utility Industry” 11th Latvian International Mathematical conference, April 1–2, 2016, Daugavpils, Latvia.
14. An. Matvejevs, J. Fjodorovs Algorithm for Imitation of a Time Span for Reaching a Border of a Random Process” RTU 57th International Scientific Conference, October 14–18, 2016, Riga, Latvia.

List of publications

1. Fjodorovs J., Matvejevs A., Malyarenko A. “Algorithms of the Copula Fit to the Nonlinear Processes in the Utility Industry”// ICTE 2016, December 2016, Riga, Latvia. Iekļauta Scopus datubāzē.
2. Matvejevs A., Fjodorovs J. “Estimation of Semi Parametric Markov Models with Frank Copula”// Proceedings of Journal of Applied Mathematics, 2015 – 292.–299. lpp. Iekļauts Scopus datubāzē.
3. Fjodorovs J. “Simulation of Option Prices Using GARCH Processes for Autocorrelated Stock Returns”// Proceedings of Journal of Applied Mathematics, 2014 – 151.–158. lpp. Iekļauta Scopus datubāzē.
4. Fjodorovs J. “Copula Estimation for GARCH(1,1) Processes”// Proceedings of Journal of Applied Mathematics, 2013 – 111.–120. lpp. Iekļauts Scopus datubāzē.
5. Matvejevs A., Fjodorovs J. “Evaluation of Dynamics of the VIX Index Via Heston Model”// Proceeding of the International Conference on Business Intelligence and Financial Engineering ICBIFE’2011, Honkonga, Hong Kong, 12.–13. decembris, 2011. – 125.–131. lpp. Iekļauta VERSITA un EBSCO datubāzēs.
6. Egle A., Matvejevs A., Fjodorovs J. “The Evaluation of Financial Assets with Autocorrelations in Returns”// Scientific Journal of Riga Technical University, 2012. – 116.–119. lpp. Iekļauta VERSITA un EBSCO datubāzēs.
7. Fjodorovs J. “Copula Based Semiparametric Regressive Models”//Journal of Applied Mathematics, 2012, Volume V, pp. 241–248, ISIN: 1337-6365. Iekļauta VERSITA un EBSCO datubāzēs.
8. Matvejevs A., Fjodorovs J. “Modeling VIX Index Based on Semi Parametric Markov Models with Frank Copula”//Scientific Journal of Riga Technical University, 2014 – 106.–110. lpp. Iekļauta VERSITA un EBSCO datubāzēs.
9. Matvejevs A., Fjodorovs J. “Revaluation of Estimated Option Prices Using GARCH Processes with Most Preferable Properties”//Scientific Journal of Riga Technical University, 2013 – 100.–104. lpp. Iekļauts VERSITA un EBSCO datubāzēs.
10. Matvejevs A., Fjodorovs J., Pavlenko O. “Testing Heston Model Consistency and Evaluation of Parameters Thought Reprezentation in Discrete Time”// Journal of Applied Mathematics, 2011, Volume IV, pp. 265–272, ISIN: 1337-6365. Iekļauts VERSITA datubāzē.

Work structure

The paper consists of Introduction, four sections, Conclusions and appendixes. The paper contains 134 pages, 40 figures, 7 tables, and 94 titles in the reference list. The structure is as follows.

The introduction is devoted to justification of the importance of the conducted research, formulation of research purpose and tasks, listing methods employed in the work and description of novelty and meaningfulness of the obtained results, as well as approbation of the results.

In the first section, “Financial time series and serial correlations”, a quantization algorithm for Heston model is developed. It may be applied to transformation of different stochastic models in discrete time. Then the stationarity and data fit of these models can be analyzed with known methods that are employed for studying ARIMA-class models. Having applied the proposed method to US stock option volatility indices VIX, one can see that model results are in line with the actual data before a decision of a buy/sell order is made. Additionally, way of obtaining data on the research objects, forecasting essence, and definition, types and stationarity of ARIMA model is described in this section.

The second section, “Non-parametric regressions and their setting using copulas”, provides the definition of a non-parametric Markov model and development of transition densities discovery method for this model using Archimedean type copulas. With this method, it is possible to convert the Markov model in copula space (where Markov chain is compact), which, in turn, allows to use the limit theorem of probability theory and approximate differential equations with diffusion equations. As a result, multiple formulas for price determination of financial instruments can be reviewed and modified based on financial markets data. For illustration purposes, Markov copula-type non-parametric regression for VIX index data and a non-linear process, which is a real-life example from energetics, is presented in this section. Moreover, a copula for GARCH(1, 1) model oscillation is created based on several conditions regarding the model’s parameters and marginal distribution.

In the third section, “GARCH(p, q) model with autocorrelated residuals and option revaluation equation”, GARCH(1, 1) model residuals are converted using Markov model with divided correlation coefficient. Black-Scholes option pricing model and the related option risk sensitivities (Greeks) is transformed. The obtained formula enables us to estimate option prices more precisely, accounting for thick tails of asset returns distribution. To solve the listed problems, we used imitation modeling in *Matlab Simulink* software.

The fourth section, “Use of autocorrelation for determining option price”, presents an overview of algorithms proposed in the work based on estimation of Tesla Motors Inc stock option prices by conducting Monte-Carlo simulations of expected volatility and option repricing. The constructed system allows forming a personal view on situation in financial markets and particularly optioning prices, and making a justified decision to enter long or short positions in certain options.

Results and conclusions.

1. DEFINITION OF THEORETICAL MODEL AND DESCRIPTION OF RESIDUAL AUTOCORRELATION

One of the main contemporary problems in econometrics is the analysis of $\{x_t, t \in Z\}$ time series using autoregressive models without any a priori assumptions regarding the expected value of the phase coordinate conditional on the previous observation in the time-series data.

The underlying reason for the tendency to depart from traditional linear models is the non-Gaussian character of random values that describe real model behavior.

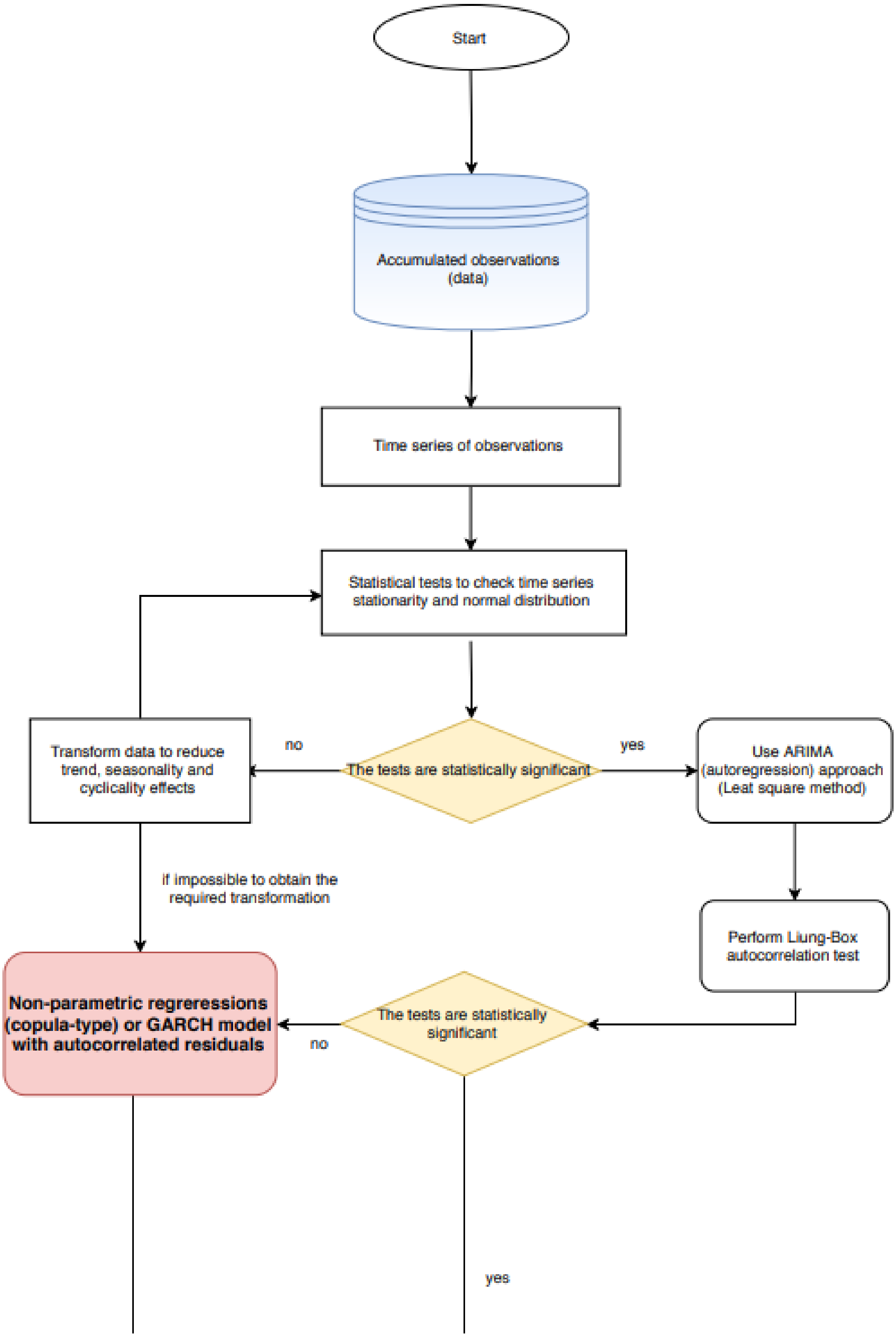
The main aspect of time series analysis is study, description and/or modelling of data structure [3]. The purpose of such studies usually is not limited to the analysis and modeling of existing processes. In most cases, the constructed model is employed for extrapolation or forecasting of time series data. Thus, the quality of such forecast may serve as a good measure for construction of alternative models. Creating a good model is also necessary for other applications like correction of seasonal effects and smoothing [4]. Finally, these models can be used for statistical modeling of large systems where the input data is in time series form.

Measurement errors in economic indicators and the non-avoidable presence of random fluctuations makes using probability theory and mathematical statistics approach for time series analysis ubiquitous. These sciences interpret the observable time series data as a realization of a random process. It is generally assumed that the data possesses a certain structure, in this way differing from a series of independent random values because observations are not a set of fully independent values. Some structural elements of time series, such as trends and periods, can be discovered straight away by simply looking on plotted values. Usually, it is assumed that the structure of a series can be described with a model that contains a small number of parameters comparing to the number of observations in the series. It is important in practical applications to be able to use that constructed model for forecasting. Examples of such models are autoregression model AR(p), moving average model MA(q), as well as their combinations – models ARMA(p, q) and ARIMA(p, k, q). [3]

In most research papers, a side takeaway from analysis is modeling of macroeconomic indicators that describe inflation, international trade, interest rates, exchange rates and other processes, there is observation of similarities in behavior of random residuals (forecast errors): their huge and small values form concentrated groups, i.e. steady and volatile periods go after each other [7]. Moreover, these series do not cease to be stationary and homoscedastic (i.e. error distribution is homogenous) even during long time periods, so the hypothesis of constant variance is not violated by the actual data [8]. ARMA model does not help to solve this phenomenon. Therefore, a modification of models known at that time is needed.

The first mention of such modification is by R. Engle [6], who observes residuals as conditionally heteroscedastic, i.e. related to autoregressive dependency. He developed ARCH(q) model, which had plentiful modifications, inter alia the most popular GARCH(p, q).

Information scarcity about the distribution of data does not allow to calculate the abovementioned expected value analytically, in a functional form with unknown parameters, and reduce the problem to the least squares estimation method, as is common for Gauss' theory (Fig. 1.1.).



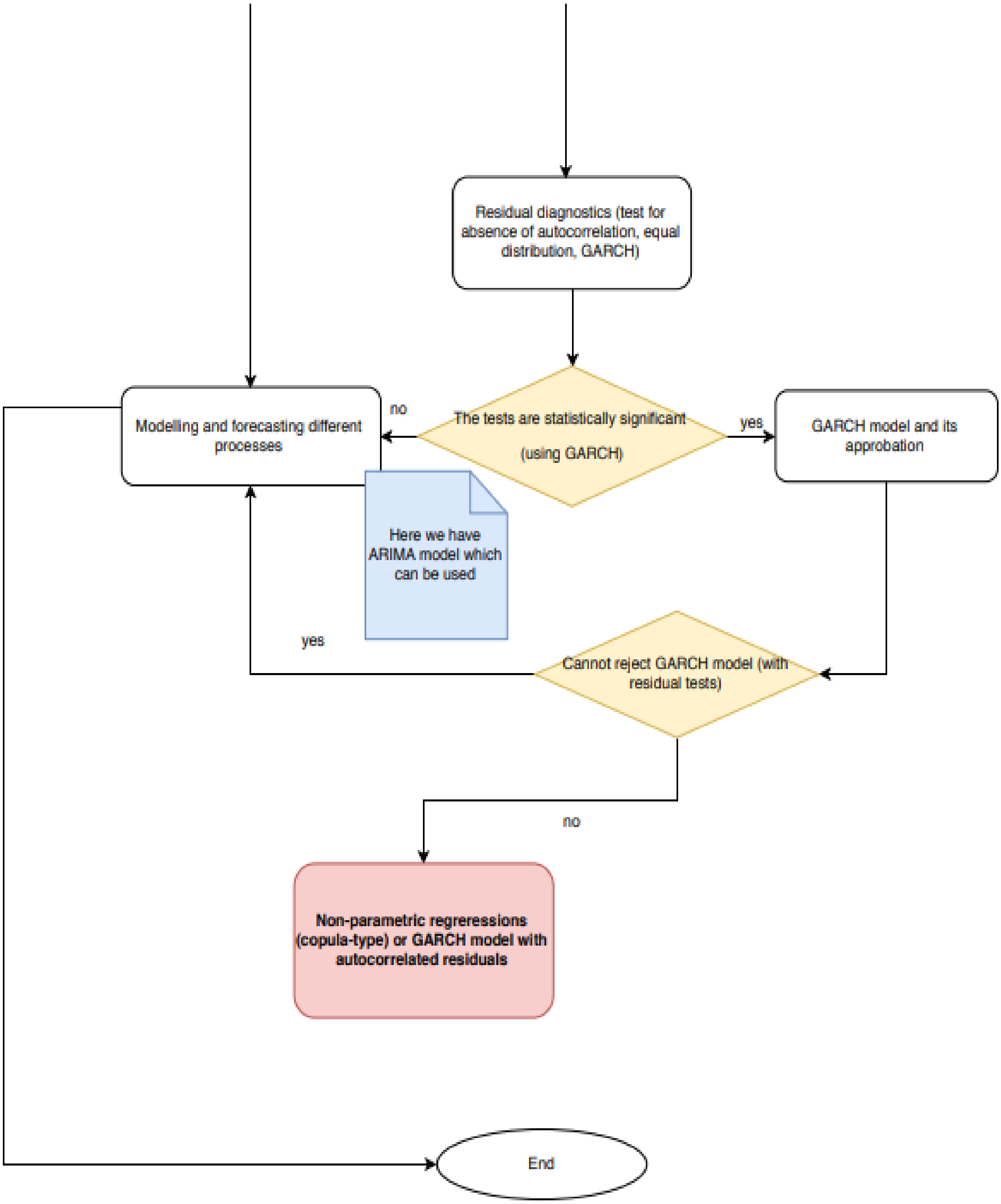


Fig. 1.1. Algorithm for constructing statistical models.

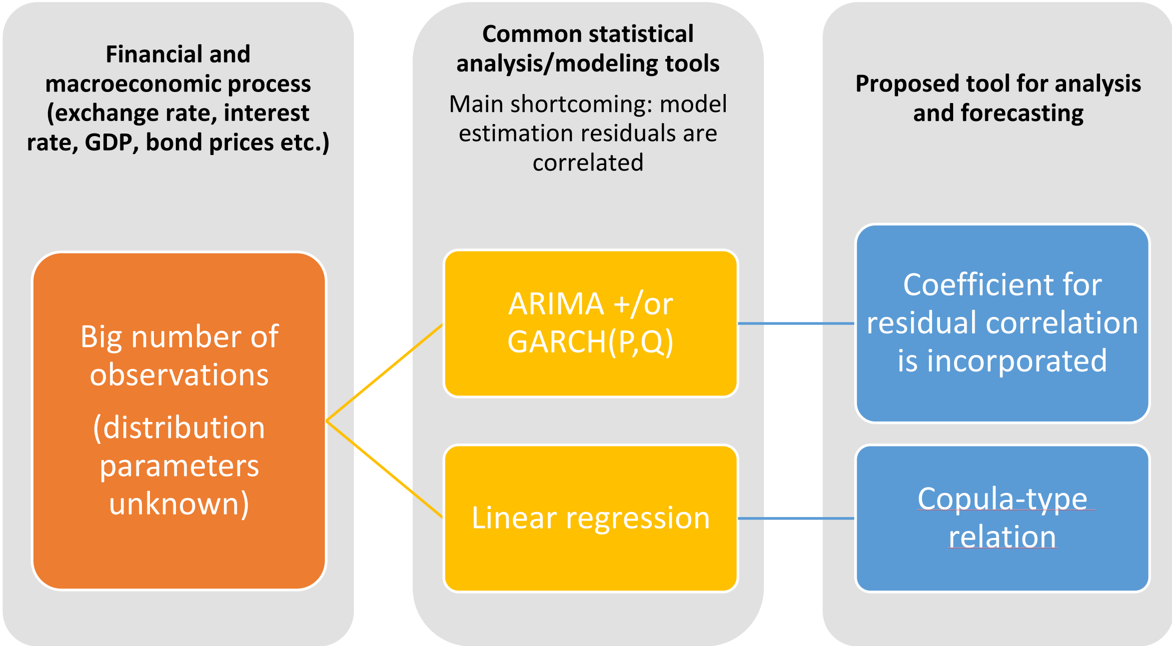


Fig. 1.2. Data processing models.

Figure 1.2 presents existing models (in the center) and additions to these models developed in the work. These additions help to improve imitation and forecasting abilities of different processes, i.e. allow to describe processes with residuals with serial correlation (by introducing the coefficient of residual correlation), as well as create models (based on copulas of different types) that describe not only normally distributed data, but also those with "thick tails" and a "spiked peak". This approach in model imitation[1] allows predicting rare outcomes more often than the normal distribution. Thus, various calculations in finance, economics, and energetics are improved.

Heston model and its transformation to discrete time

The goal of this work is to present an overview of applications of ARIMA model in order to conduct a representation of continuous Markov processes in the case of discrete time. It is known that each Markov process can be interpreted as a limit case of a discrete Markov process. Similarly, the solution of Kolmogorov diffusion equation can be approximated by solutions of Kolmogorov differential equations [25].

From this point, the Heston Volatility Model is described. This method is designed to give an intuitive explanation of the Heston model, not only technically, but also in a way that makes the subsequent sections more easily understandable. Should any questions regarding smaller technical details arise, the answers can be found following the provided references.

[1] The author uses the word "imitation" meaning the Monte Carlo simulation, i.e., the development of models in a certain form that could provide better results for describing and predicting the process.

Heston [71] offers the following model:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^1, \tag{1.1}$$

$$dV_t = k(\theta - V_t) dt + \sigma \sqrt{V_t} dW_t^2, \tag{1.2}$$

$$dW_t^1 dW_t^2 = \rho dt,$$

where $\{S_t\}_{t\geq 0}$ and $\{V_t\}_{t\geq 0}$ are respectively prices and oscillation processes; $\{W_t^1\}_{t\geq 0}$, $\{W_t^2\}_{t\geq 0}$ are correlated Brownian movement processes (with correlation parameter ρ); μ is a deterministic risk-free rate; $\{V_t\}_{t\geq 0}$ is the square root of the average value of the reverting process the first mention of which can be found in the work by Cox, Ingersoll, and Ross [27]; θ is the long-term mean value; σ is standard deviation; k is mean-reverse coefficient; and ρ is defined as diffusion oscillations. All parameters k, μ, θ, σ are homogenous in time and state.

Therefore, a representation of a discrete Heston model should be found with a respective fitting/testing method of an ARIMA-class model. Thus, the hypothesis about the existence of the heteroscedastic effect in model residuals can be tested using time series technique.

Modeling VIX index with Heston model

Let us consider a case where a Heston model in discrete representation is combined with an ARIMA model fitting/testing method. The object of our attention is VIX – stock market volatility index. VIX is a market instrument that evaluates the uncertainty implied in the base index, S&P 500, for 30 days in the future. The dataset contains daily observations from March 27, 2007, to October 21, 2010. The ability to correctly interpret VIX changes and reactions of its price to market events can equip investors with markedly better possibilities to manage risk/return characteristics of their portfolios, as well as develop strategies using derivative instruments based on VIX positions. As a result, this allows to maximize investor's return from S&P 500 and VIX correlations, which changes through time.

Fig. 1.3. Historical dynamics of VIX stock index.

Figure 1.3 shows that the time series data of VIX index value clearly possesses non-stationarity properties. Moreover, some trend changes after October 2008 are also obvious. Therefore, to smooth out the data, the first difference should be used. This new series DVIX is conditionally heteroscedastic. It does not contain the unit root based on Dickey-Fuller test results. Based on autocorrelation results, an appropriate model can be chosen – AR(1) with residuals modeled by ARCH(2). In turn, time series stability test (Chow breakpoint test) indicates a structural break dated October 2008. Consequently, I decide to analyze the data only after October 2008. For this purpose, a new time series is created.

We can clearly see a trend in the new time series. Hence we further create another series, *vix*2, with the trend being excluded. The time interval for the series is from October 10, 2008, to October 21, 2010.

$$vix2_t = vix_t - vix_{t-1}. \tag{1.3}$$

The system obtained in this work from Heston model discretization is approximately equal to AR(1) model with GARCH-M(1, 0) residuals, where the squared variance is included in the main AR(1) equation for $vix2_t$ with a fixed coefficient of 0.5.

Using WINRATS software package, parameters for time series $vix2_t$ are estimated in the following way:

$$\begin{cases} vix2_t = -16,743 + 0,5\,V_t^2 + 0,954 \cdot vix2_{t-1} + \varepsilon_t; \\ \varepsilon_t = v_t \sqrt{V_t}; \\ V_t = 5,9934 + 0,04138979 V_{t-1}. \end{cases} \tag{1.4}$$

The discrete Heston model representation is appropriate for solving the problem related to correspondence of the stochastic model to a financial time series. Moreover, employing ARIMA method for estimation of Heston model parameters, makes the process much easier, i.e. model discretization simplifies the estimation of problem parameters. However, considering the abovementioned equation coefficients, VIX data modeling and respectively forecasting future values of the index using the Heston model may not always be correct. Therefore, the approach of stochastic Heston model discretization can be used for testing the fit of stochastic differential equation to the real data (Figs. 1.4 and 1.5).

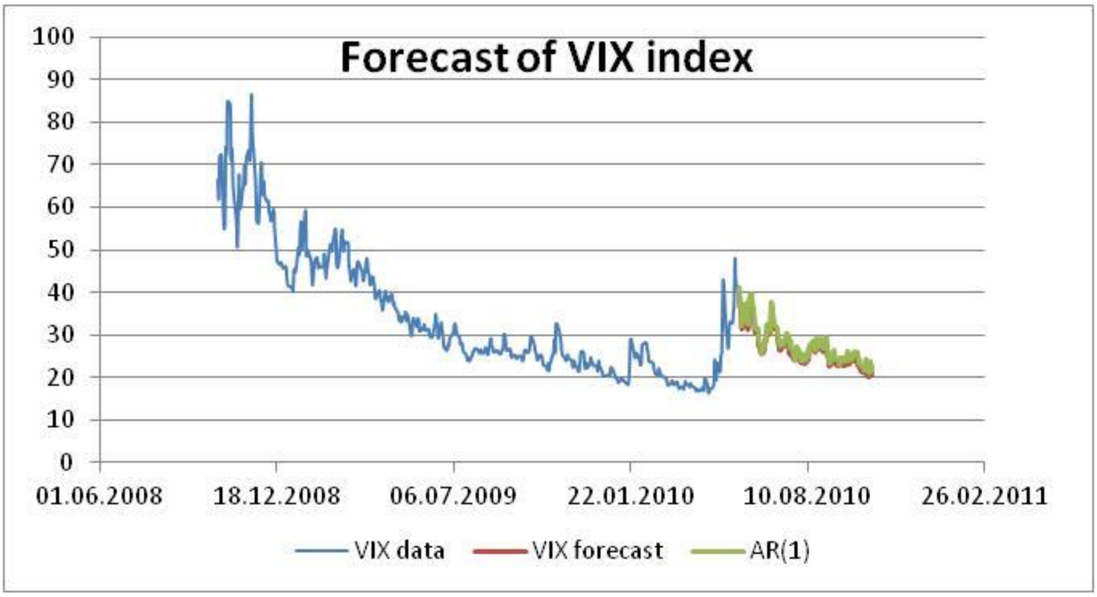


Fig. 1.4. VIX index and its forecasted values.

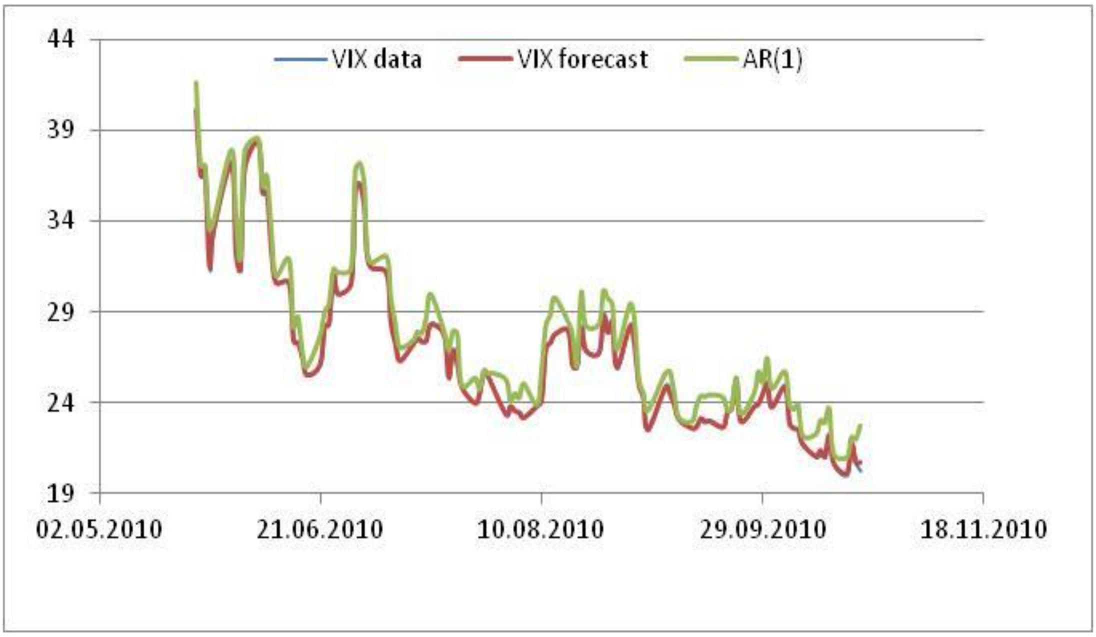


Fig. 1.5. VIX index and comparison of forecasts made by different models.

Hence, one can conclude that model transformation to the discrete time allows to determine model fit to the real data, which increases forecasting accuracy.

Summary of the first section

1. The class of autoregressive models (ARIMA) does not fully describe all processes in mechanics and economy. Therefore, it makes sense to include correlation parameter residuals to effectively reduce forecasting errors.
2. An algorithm of Heston model transformation is created to improve forecasting accuracy for heteroscedastic processes.

2. NON-PARAMETRIC REGRESSIONS AND THEIR SETUP USING COPULAS

Non-linear time series can be identified using estimations of the conditional mean value and the conditional variance, which is mentioned in plentiful articles (see, for instance, [31]). Usually, the dependency of regressive models of stationary time series data $x_t, t \in Z$ is analyzed, which generates time process relation. As demonstrated in [31], this allows to separate time dependency (for example, tail relation) from the marginal distribution of time series behavior (such as heavy tails). Another advantage of this method is the ability to apply the limit theorem from probability theory for transition from an equation in differences to the continuous stochastic differential equation [32], [33].

In this work, a class of copulas is studied based on non-parametric stationary Markov model in the form of a scalar differential equation:

$$t \in Z : X_t = X_{t-1} + \varepsilon f(X_{t-1}, \varepsilon) + \varepsilon g(X_{t-1}, \varepsilon)\xi_t, \tag{2.1}$$

where $\{\xi_t, t \in Z\}$ is i.i.d. (identically independent distributed) $N(0, 1)$ and ε is a small positive parameter, which is used for the approximation of regression (2.1). The regression is frequently used for imitations and estimation of the parameters in the stochastic oscillations model [32]. Unfortunately, a Markov chain set by Equation (2.1) is not compact in the phase space, which hinders employing the limit theorem of probability theory. Copulas based methodology lets us simplify the asymptotic analysis of Equation (2.1).

Let us recall that to construct copula $C(u, v)$ for the pair $\{X_{t-1}, X_t\}$ from Equation (2.1), it is necessary to find the marginal invariant distribution of X_t, $F(x)$, and insert it into the joint distribution function $H(x, y) = P(X_{t-1} \le x, X_t \le y)$, where $C(u, v) = H(F^{-1}(u), F^{-1}(v))$ and $H(x, y) = C(F(x), F(y))$. Considering the existence of a small parameter ε after substituting $U_t = F(X_t)$ in Equation (2.1), the diffusion approximation can further be considered a diffusion equation in the same way as in (2.1):

$$t \in Z : U_t = U_{t-1} + \varepsilon f(U_{t-1}, \varepsilon) + \varepsilon g(U_{t-1}, \varepsilon)\xi_t. \tag{2.2}$$

This makes construction of the transmission probability more tractable and function estimations $\hat{f}(u)$ and $\hat{g}(u)$ easier to obtain.

Upon the diffusion approximation in Equation (2.2), it is easier to do inverse substitution and obtain a stochastic equation, as the diffusion approximation in Equation (2.2).

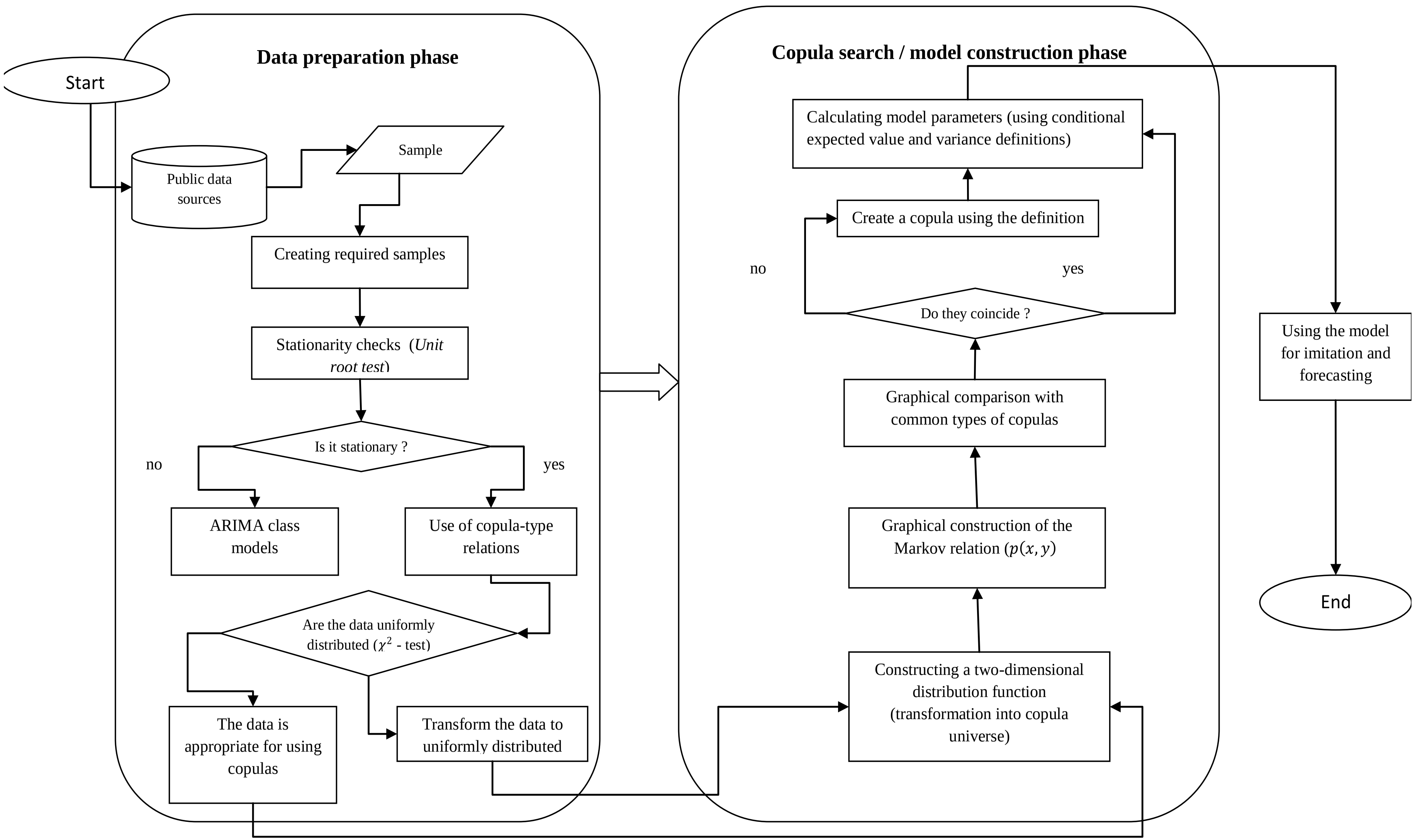


Fig. 2.1. Algorithm for constructing a copula-type regression.

Finding copulas for non-linear processes in energetics and algorithm for imitation of the reaching time of the random process defined limit

The use of algorithms for constructing copulas and copula-type regressions leads to an opportunity of finding processes with random behavior not only in finance (see Fig. 2.1), but also in the rest of economy. For example, an algorithm that helps to construct an imitation model for a problem in energetics is created in the context of this work. Such a model allows forecasting the maximum power of production facilities.

For this purpose, the work implies analysis of historical observations of operation parameters of plants (see line Y in Fig. 2.2). To achieve the effectiveness of plants' work and receive early signals about possible malfunction in the future, one can decide on a stage whereat preventive works should be implemented. We are therefore interested in stable work of plants, i.e. the low volatility of the process. In real life, however, parameters can vary significantly because of different conditions. Hence our idea is to imitate a forecast of substantial changes of parameters. Our main idea is to define the limit for allowed deviations and find and algorithm that can imitate this defined limit. Obviously, it is related to heteroscedastic processes. To model the described process Y, the ARIMA-GARCH approach can be employed. Additionally, copulas may be of use for modelling critical moments, which are expected to occur relatively seldom.

Considering the proposed task, the copula-type regression is adapted to time series Y in order to create an imitation of a certain parameter arriving at a pre-defined level, as well as an imitation of reaching the limit set by random time. Time series Y consists of daily observations from December 31, 2000, to December 31, 2015.

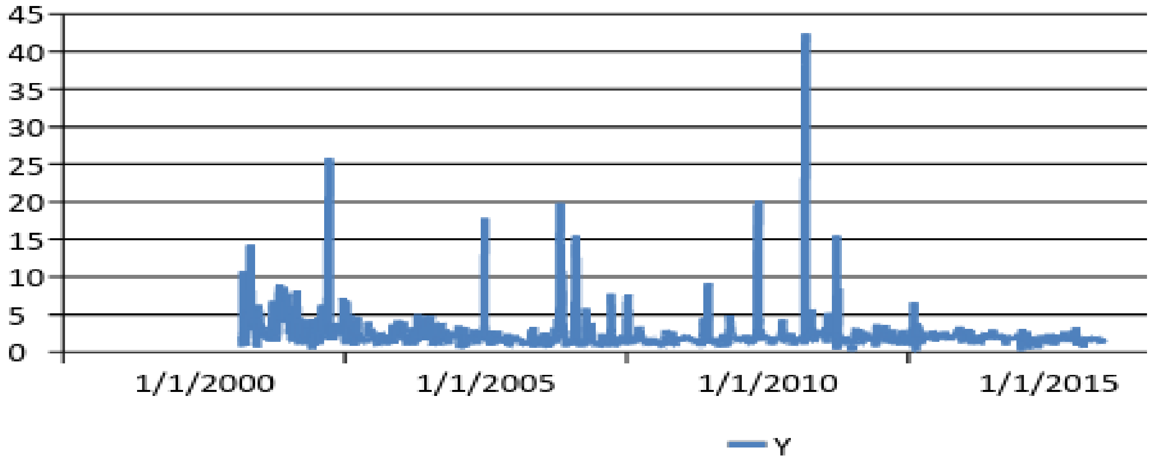


Fig. 2.2. Observations of time series Y

In turn, a copula for process Y is found using the algorithm described in Section 3 (Fig. 2.1). MATLAB software is used to construct the following density functions.

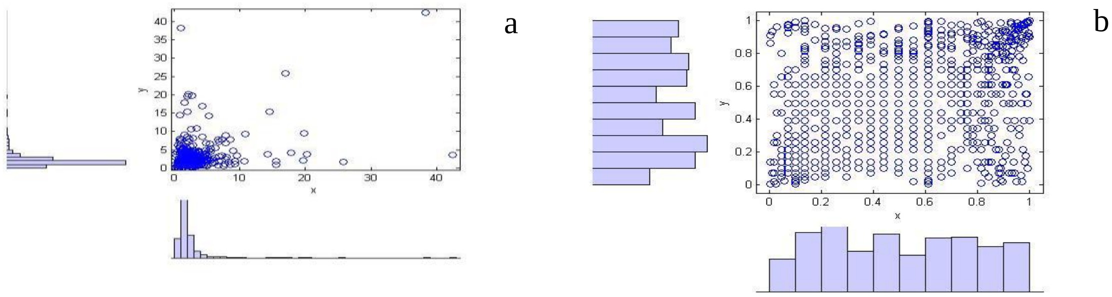


Fig. 2.3. (a) Plot of the two-dimensional function for data series Y (not transformed to *R*[0, 1]); (b) plot of the two-dimensional function for data series Y transformed to *R*[0, 1].

As it can be seen from Fig 2.3 (a), time series Y contains outliers. This significantly complicates the search of marginal distribution. Based on Kolmogorov-Smirnov test, different distributions have been tried. Eventually, the best turns out to be the joint distribution – one part of the data is well described by the exponential distribution and the other – by the uniform distribution:

$$F(x)=\begin{cases}1-e^{-\lambda x},\ x<T,\\ H(x-T)+e^{-\lambda T},\ T_1<x<T,\end{cases} \tag{2.3}$$

$$H=\frac{e^{-\lambda T_1}}{T-T_1}, \tag{2.4}$$

where T is the length of time series and T_1 is the length of time series without outliers.

Considering the constructed marginal distribution, time series data Y is transformed into the uniform distribution (*R*[0, 1]). The most important step in this study is the choice of a copula that is appropriate for the data. Several standardized copulas are reviewed in this work – the Gumbel, the Frank, the normal and *T* copula. In different articles, various approaches to checking the appropriateness of a chosen copula to the data are mentioned, such as AIC and BIC criteria, χ^2-criteria and the Kolmogorov–Smirnov distance. The primary choice of copula for this research is based on the Kolmogorov–Smirnov test (see Table 2.1).

Table 2.1

Kolmogorov–Smirnov Test (*KS* distance)

Copulas	***KS* distance**
The Franka copula	0.67
The Gumbel copula	0.65
The Normal copula	0.18
T copula	0.70

$$D_{KS}=\max_{i,j}\left|C_n(U_{1,i},U_{2,j})-C_\theta(U_{1,i},U_{2,j})\right|. \tag{2.5}$$

KS test results suggest that the best copula for time series Y is the normal copula. However, the normal copula links random variables linearly in essence. At the same time, we are interested to imitate infrequent jumps in Y, which serve as a signal for possible malfunction of plants. To satisfy this requirement, the copula that describes an asymmetrical dependency is the Gumbel copula. Based on this copula, we derive nonparametric regression coefficients (functions) *f* and *g*.

For the Gumbel copula, the inverse function cannot be used with the goal to return to the main equation. To work with copulas of such type, certain special algorithms should be employed to return to the required equation form.

Table 2.2

Algorithm for Imitation of a Time Span for Reaching the Border of a Random Process

1. Constructing Margins

$\{X_n\} \to \{Y_n = F(X_n)\}, Y_n \in [0,1];$

2. Finding Copula and it parameters

$C(y,z) = P(Y_n \le y, Y_{n-1} \le z);$

3. Estimation of semi parametric regression model via copula:

$\sigma^2(z) = m_2(z) - m^2(z);\ dz = m(z)dt + \sigma(z)dw(t)$

$m(z) = E(Y_n / Y_{n-1} = z) = \int_0^1 yp(y,z)dy$

4. Constructing iteration procedure in a points t_n with small parameter $h = 0.01$

$y(t_{n+1}) = y(t_n) + h^2 m(y(t_n)) + \xi_n \sigma(y(t_n))h,\ \ \xi_n \sim N(0,1)$ (*)

$x \in (0,\Gamma), X_0 = x, \tau(x,\Gamma) = \inf\{n > 0, X_n \ge \Gamma\} \Leftrightarrow$

$y(0) = F(x), \tau(x,\Gamma) = \inf\{t_n > 0, y(t_n) \ge F(\Gamma)\}$

5. Finding distribution of time $\tau(x,\Gamma)$ to reach $X_* = \Gamma$ via Monte Carlo imitation

5.1. Making iterations of the (*) N times until $y(t_*) \ge F(\Gamma)$ and after each iteration remember number $n^{(k)}$

5.2. Construct histogram of the $\{n^{(k)}, k = 1, ..., N\}$ and find distribution

Next, the utilized model should be checked for its appropriateness to the data and compared to the other models. However, considering the non-stationary behavior of the data (time series Y), especially in the moments when the jumps are observed, ARIMA models in their classical form cannot be applied, and the chosen non-parametric model with the Gumbel copula can be tested using an imitation.

As a result, Table 2.2 presents an algorithm for the construction of a non-parametric copula-type regression, which helps to solve random process imitation tasks (with heteroscedastic nature). This algorithm allows to imitate heteroscedastic random processes and find the time interval distribution for reaching the limit of this process. This is very important in energetics (reaching a certain limit), for example in plant operations it can help to readily prepare for possible malfunction.

The next section contains an example of imitation of time series Y using the Gumbel copula. Having constructed the algorithm for copula-type regressions and taking into account the procedure of imitation of time to reach a certain limit, which is described in Table 2.2 (steps 1 to 4), we model the process Y with the Gumbel copula and a non-parametric regression (Fig. 2.4). In essence, our imitation reacts on oscillations of the process. This allows us to use this model for estimation of time distribution.

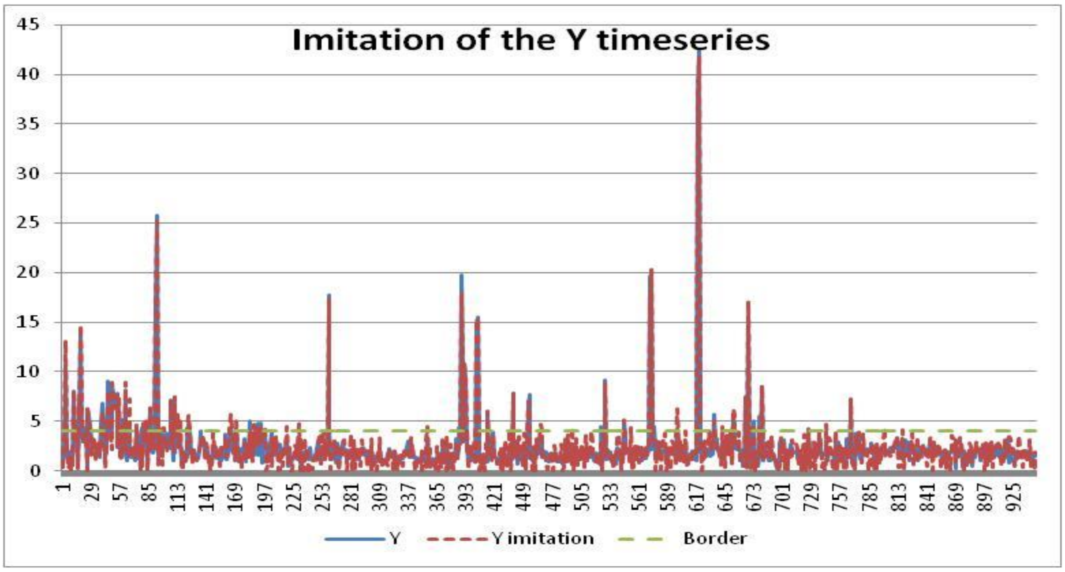


Fig. 2.4. Imitation of Y time series and defined border.

While working with copulas, certain facts should not be omitted. For example, it is difficult to establish which copula is the most appropriate for a certain dataset, because some copulas more accurately fit the data near the center of the distribution, whilst others – at the tails of a distribution. Many copulas do not have moments closely linked to the Pearson correlation, hence cannot be easily compared with financial models based on correlations.

Summary of the second section

The developed algorithm for constructing non-linear (non-parametric) Markov models based on copulas allows to model dependencies of different types – for the center and tails of the distribution. Moreover, this section describes the construction of GARCH(1, 1) copula and estimation of parameters for non-linear models. For several copulas, the inverse transformation does not exist, which diminishes the realization speed of the mentioned algorithm.

3. GARCH(p, q) MODEL WITH AUTOCORRELATED RESIDUALS AND OPTION REVALUATION EQUATION

Mezins [60] has discovered the difference between the volatility of a financial asset's price, the volatility of a financial asset's return, and the autocorrelation coefficient of a financial asset's return. His analytical solution can be reduced to a special case of the well-known Black-Scholes option pricing formula for autocorrelated returns of financial assets. Mezins' paper presents a framework for a log-normally distributed asset price S with serial correlation in returns. Additionally, he constructs an analytical option-pricing model that allows to obtain an exact solution for setting the price of a derivative instrument on a certain financial asset. The framework has been constructed for a normally distributed random process x, for which $\ln S = X$ and whose autocorrelated increments ξ have volatility coefficient σ^2 and autocorrelation coefficient ρ. Both parameters can be estimated from historical data. If the heuristic form is avoided, it is possible to utilize limit theorems proposed by Carkovs [50] and obtain an approximation for a stochastic differential equation for continuous time expressed as a diffusion approximation.

Further, one of the methods of using asymptotical methods in real life is described. Particularly, they can be applied for repricing of options and the corresponding sensitivity parameters.

A simple mathematical model that describes changes of stock price S and implies an assumption about serial autocorrelations in stock returns can be expressed in the following form in the case of a usual assumption regarding a risk-neutral probability measure P:

$$S_{t+1} = S_t \left(1 + \varepsilon^2 \mu + \varepsilon \sigma y_{t+1}\right), \quad (3.1)$$

where y_t is a Gaussian-type series with the zero mean and unit variance.

If independence of observations is assumed, y_t can be expressed as an AR(1) process (Markov process):

$$y_{t+1} = \rho y_t + \sqrt{1-\rho^2}\,\xi_{t+1}\,, \quad (3.2)$$

where $\{\zeta\}_t$, $E\zeta_t$, = 0, $E\zeta_t^2 = 1$ are independent and equally distributed Gaussian series.

To use the results of Carkovs [50], let us denote the result x_t: = S_t and rewrite Equation (4.1) as follows:

$$x_{t+1} = x_t + \varepsilon \sigma y_{t+1} x_t + \varepsilon^2 \mu x_t. \quad (3.3)$$

As follows from this result, for small ε Equation (3.3) can be approximated with vector distribution$\{X(t_1), X(t_2), \ldots, X(t_n)\}$, which is defined with the help of the following Ito stochastic differential equation solution:

$$\mathrm{d}X(s) = a\big(X(s)\big)\mathrm{d}s + \sigma\big(X(s)\big)\mathrm{d}\omega(s). \quad (3.4)$$

Solving (4.4) we obtain stochastic differential Equation (4.3) in the form of diffusion process, which satisfies Ito stochastic differential equation

$$\mathrm{d}S(t) = S(t)\big(\mu + \sigma^2 k\big)\mathrm{d}t + S(t)\sqrt{1+2k}\,\sigma \mathrm{d}\omega(t), \quad (3.5)$$

$$k := \sum_{m=1}^{\infty} \mathrm{Corr}\{y_{t+m}, y_t\} = \frac{\rho}{1-\rho}.$$

By making the substitution, we obtain the ultimate equation:

$$\mathrm{d}S(t) = S(t)\left(\mu + \sigma^2 \frac{\rho}{1-\rho}\right)\mathrm{d}t + S(t)\sqrt{\frac{1+\rho}{1-\rho}}\sigma \mathrm{d}\omega(t). \quad (3.6)$$

Option repricing equation

Based on the obtained results, European-type call options for stocks with autocorrelated returns can be repriced, and market risk sensitivity parameters can be estimated.

Let us proceed with expressing a formula for the price of European call options in the case when stock price process $S(t)$ satisfies stochastic differential Equation (4.6). The limit conditions of a European call option are $C(S(T), T) = \max(S(t) - K, 0)$, and $C(0, t) = 0$. Using the standard approach, we obtain

$$C(S(t), t) = S(t)N(d_1) - K\exp\left(-\left(\mu + \sigma^2 k\right)(T-t)\right)N(d_2), \quad (3.7)$$

where

$$d_1 = \frac{\log\left(\frac{S(t)}{K}\right) + \left(\mu + \sigma^2 k + \frac{1}{2}\sigma^2(1+2k)\right)(T-t)}{\sigma\sqrt{(1+2k)(T-t)}},$$

and

$$d_2 = d_1 - \sigma\sqrt{(1+2k)(T-t)},$$

where $N(d_1)$ and $N(d_2)$ are the cumulative functions of the normal distribution.

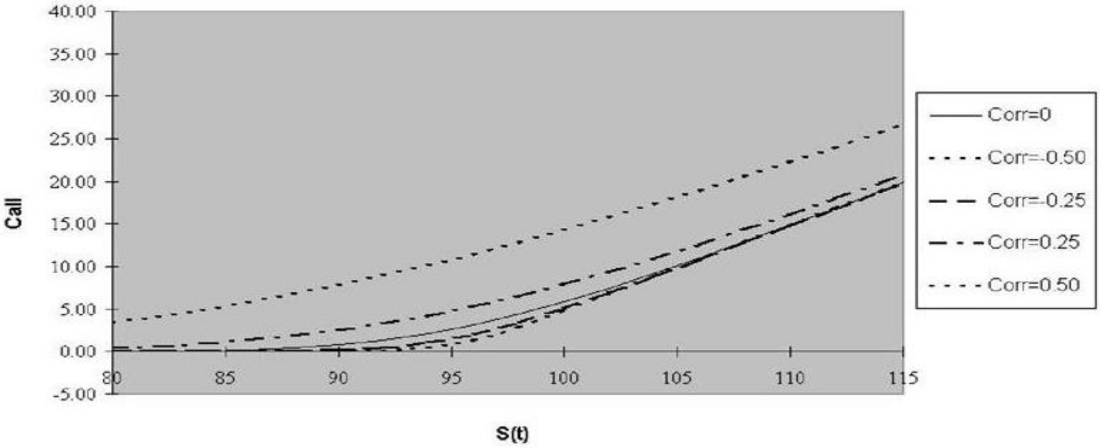


Fig. 3.1. *Call* option price for different correlation coefficients.

Recalculation of option risk sensitivity parameters (Option Greeks)

Now it is possible to obtain formulas to calculate option price sensitivity to changes of the basic parameters. There exists a number of risk measures, which are usually denoted by letters of the Greek alphabet. To illustrate the autocorrelation problem, the most commonly used are Delta, Vega, Rho and Gamma.

These measures are very important in risk management. Each measure shows the change of option portfolio value due to the impact of different market factors. Thus, a portfolio can be rebalanced with respect to a certain risk component (interest rate, exchange rate, stock price, etc.) to obtain a specific explicit position against the given risk. These indicators can be easily calculated using the Black-Scholes formula – therefore they are very important for traders of financial instruments, especially those who want to limit the risk of portfolio price changes against certain changes in financial markets. Those risk-limiting indicators that measure the sensitivity to price changes of the underlying asset are mostly used, as well as time to maturity and volatility of the derivative instrument. Risk indicators against Delta, Vega and Gamma are not widespread. Moreover, portfolio managers do not usually account for option price sensitivity with respect to risk-free interest rate changes because this risk is immaterial. All the Greeks (Delta, Vega and Rho) used in this paper are first and second-order derivatives from option prices.

Delta, Δ, is the first derivative of European call option price *C* by the underlying asset price S and is the measure of option price against the changes in the underlying asset:

$$\Delta(S(t),t) = \frac{\partial C}{\partial S} = N(d_1)$$

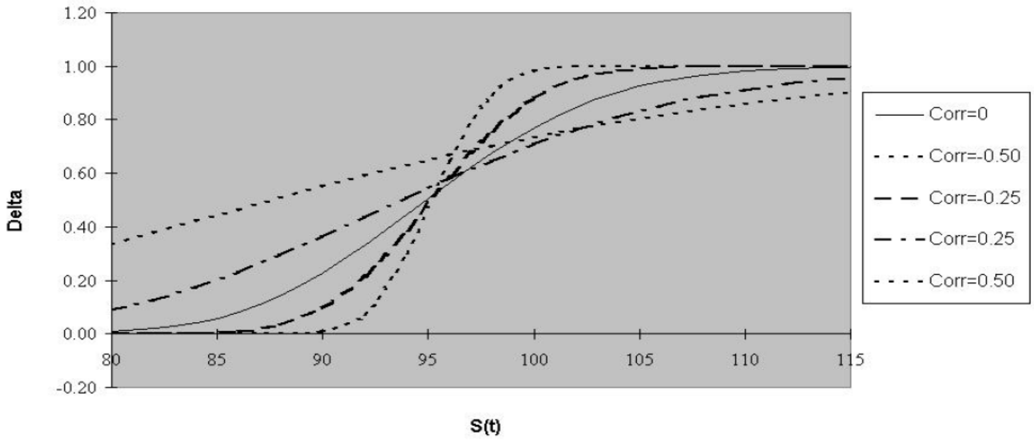


Fig. 3.2. Call option of contract Delta parameter value for different correlation coefficients.

As we see from Figs. 3.1 and 3.2, the option price is substantially affected by autocorrelation of returns of the underlying asset. If this autocorrelation exists, then the writer of an option contract may overestimate or underestimate (depending on the sign of the autocorrelation coefficient) the market risk, which can cause unexpected losses or even lead to solvency issues for the financial institution. The rest risk measures are described in the full text of the Doctoral Thesis (autocorrelations of the underlying asset have a huge role here, too).

Testing of stationary solution distribution function and calculation of time to convergence for fixed parameter values

The next step included in the Doctoral Thesis is the stability test for the solution of Equation (3.4) and respectively the obtained Equation (3.5). If it is not possible to obtain a stationary solution for Equation (3.5) and demonstrate that it is independent from the value of the correlation coefficient, then the obtained formula for calculation of the European call option price and risk measures (Greeks) does not possess a stable solution, but there exists a local solution that cannot be utilized.

$$T_\varepsilon(\rho) = \frac{\ln(\delta\varepsilon) - \ln 4\left(E\{|z(0)|^2\}\right)}{\lambda_2(\rho)}. \tag{3.8}$$

Formula (3.8) describes the time to convergence to stationary solution $\hat{x}(t)$. Thereby, using (3.8), which depends on ρ, it is possible to find the time to convergence. Moreover, starting from the convergence moment, the stationary solution should follow Gamma distribution.

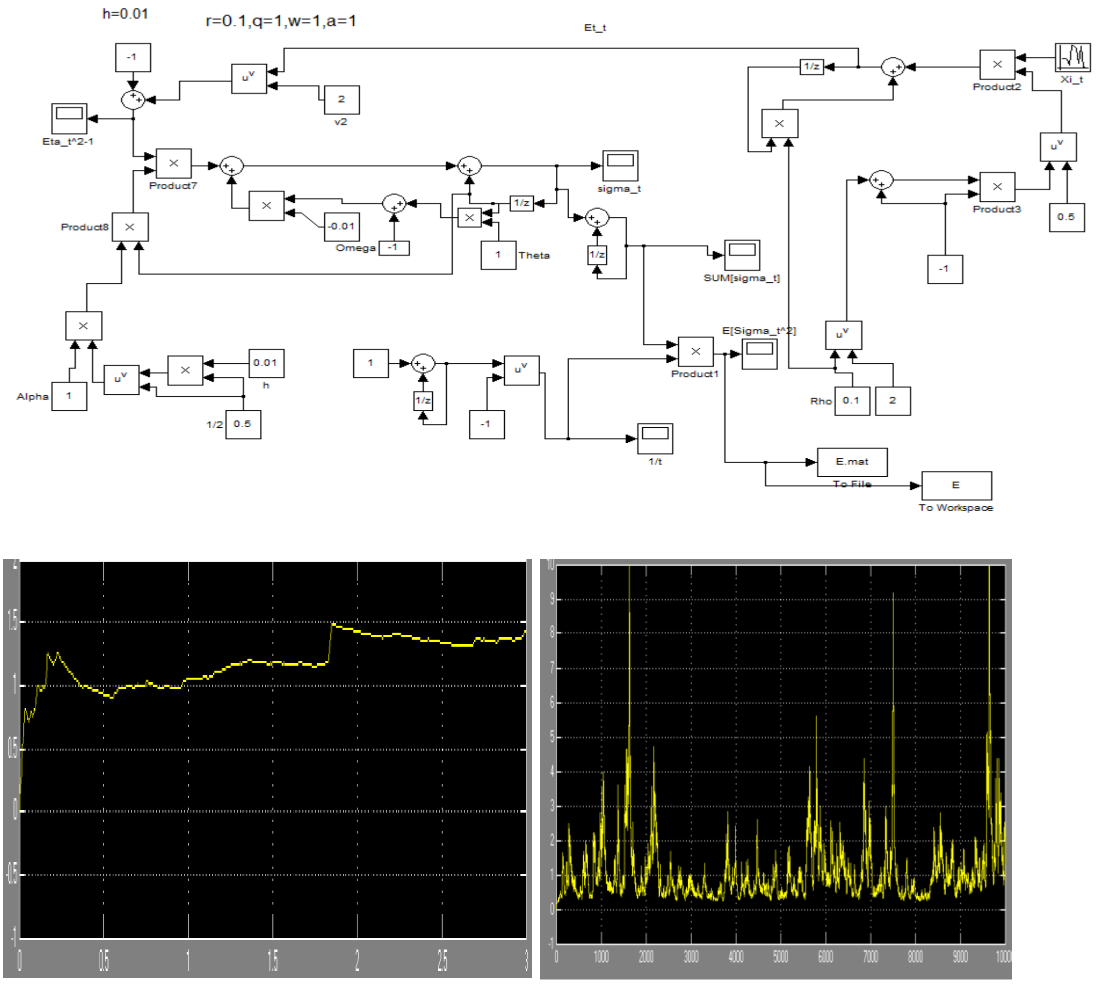


Fig. 3.3. Imitation of $E\sigma^2$ and σ^2processes in Matlab Simulink environment.

To conduct a stability analysis of the stationary solution for different correlation coefficient values and study their fit with the Gamma distribution, an imitation of theoretical

Equation (3.5) was performed (see Fig. 3.3). From 5000 observations, time series were created to calculate and compare the theoretical and empirical moments, perform the Kolmogorov test for the Gamma distribution, as well as find the time to convergence for Equation (3.5).

Table 3.1

Comparison of Empirical and Theoretical Moments

Moments	**Empirical σ^2**		**Theoretical σ^2**
	5000 imitations	**Last 100 imitations**	
Average	1.00493	0.99748	1.00010
Variance	0.00570	0.00459	0.00513

As a result, as we can see from Table 3.1, moments of different orders have no significant differences from the Gamma distribution.

In this case, the parameters of Equation (3.8) and Gamma distribution have been fixed. Imitations have been performed by changing ρ values from 0 to 1, which resulted in varying time to convergence.

Considering Nelson's [34] results, which imply that stationary solutions must be distributed following the inverse Gamma distribution, the Kolmogorov test was performed to test the hypothesis about the Gamma distribution. In our view, these tests serve as an empirical proof of (3.8). In the majority of cases, the hypothesis about the Gamma distribution has not been rejected.

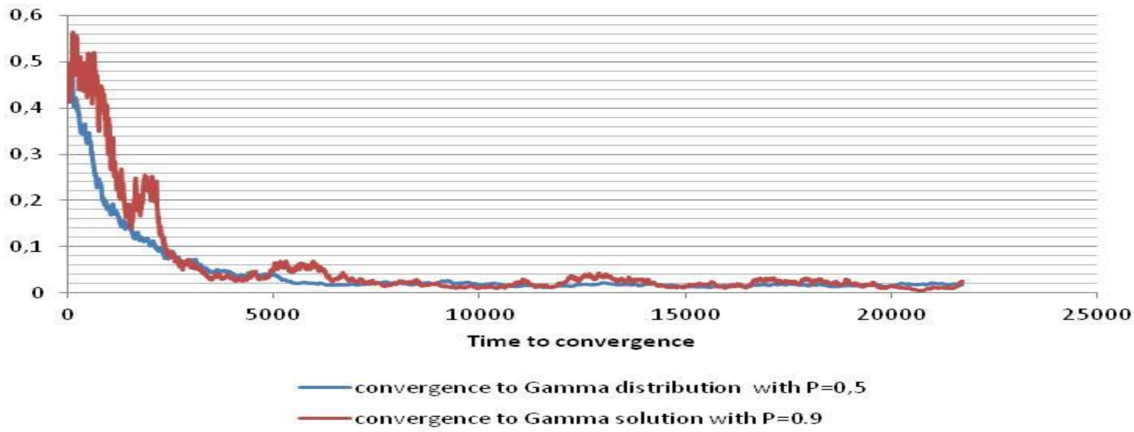


Fig. 3.4. Time to convergence to the Gamma distribution for different correlation coefficients.

As we can see from Fig. 3.4, for the correlation coefficient value of $0.5|z(t)|^2$ converges to a stationary solution with a pre-defined precision of $\varepsilon < 0.1$ faster than for the value of 0.9, however, the convergence has been obtained for all ρ values. This means that Equation (3.7) can be used to estimate the price of a financial asset with autocorrelated returns and set the respective values for risk measures.

Summary of the third section

A continuous time diffusion model for stock returns with serial correlation is developed in this work. Next, a formula for European call option pricing is obtained for the case when an option contract is written for a stock with autocorrelated returns. It is demonstrated, that even a small serial correlation creates partial predictability and leads to substantial deviations from the results of the Black-Scholes formula. Furthermore, the formulas are expanded for sensitivity parameters of the European call option price, and it is shown that hedging parameters widely used in financial risk management are dependent on assumptions about return correlations of the underlying asset. This approach can be used also for discrete time differential equations in the cases when volatility is stochastic or modelled with a GARCH process.

4. USE OF AUTOCORRELATION FOR DETERMINING OPTION PRICE

Last year, extensive discussions about an option with a very small execution probability (deep out of money) took place [44]. Some experts suggest that insuring some asset prices (writing option contracts) and selling lottery tickets provides positive investment returns in various investment sectors. There exist plentiful strategies in financial markets that describe insurance and lotteries. The main question is whether investors can improve their long-term earnings by buying or selling insurance and lottery or, respectively, using financial transactions with insurance or lottery return structure. The answer is contingent on market price asymmetry or from whether investors evaluate price asymmetry relative to the mean value. Price fluctuations both in the left tail (insurance) and the right tail (lottery tickets) increase long-term returns. However, insuring low-probability risks of option contracts and holding lottery-type investments with high volatility lead to lower long-term returns.

Further, we would like to suggest a way [46], [47] of analyzing gains and risks from option contracts based on the non-linearity of low-probability regions (GARCH model) using stock price dynamic of Tesla Motors Inc. as an example. The main idea is to use autocorrelation effect in the stochastic differential equation (expressed through a diffusion approximation with stochastic volatility described in GARCH(1, 1) form) approaching continuous time.

Table 4.1

Repricing of an Option of Tesla Motors, Inc

Option price	**Black–Scholes equation**	**Studied**
Maturity	1 year	1 year
Strike	380 USD	380 USD
Volatility	57 %	35 %
%	1 %	1 %
Price	5.22 USD	1.36 USD; if $\rho = 0.2$
		102.5 USD; if $\rho = 0.9$

A security portfolio manager, risk analyst or any other person selling option contracts can use the results of Table 4.1 in the decision-making process. As we see from the table, the new estimation of Tesla Motors Inc. stock price with a low autocorrelation coefficient is lower than offered in stock market. This result is invalidated if a higher value of the coefficient is used. In the case of a small autocorrelation coefficient, Equation (4.7) with volatility estimation using GARCH(1, 1) process suggests to sell European call option of Tesla Motors Inc., since its price is valued too high by markets.

RESULTS AND CONCLUSIONS

The main purpose of the Thesis was to develop forecasting methods using stochastic models with Markov switches. The objective of the method is to construct models accounting for residual correlations and considering "heavy tails" and "high peaks" in sample distributions. Such non-parametric model construction method using conditional moments of copula density function has huge potential for application in forecasting of financial, macroeconomic and insurance time series. Isolating correlations in model residuals and their further evaluation helps to estimate prices of derivative instruments more accurately. To reach the main goal of the promotion work, the following tasks have been proposed:

- to develop a method of constructing autoregressive model based on copula-type relations;
- to develop a method for incorporating the balance correlation into a stochastic differential equation and a transition to continuous time in order to study the stationary of this equation;
- to introduce a correction for correlation in the D. Nelson's conditional variance risk management model in the discrete time;
- to analyse the impact of correlation of observation errors on the Black-Scholes model parameters and recalculate the hedging factors.

The aim of the promotion work has been achieved and the objectives of the promotion work were fulfilled.

1. A non-parametric Markov model has been defined and a method for finding other densities of this model using Archimedes type couplings has been developed.
2. Using Markov model with a separated correlation coefficient, GARCH(1, 1) model balances have been modified and the method of incorporating balance correlation has been introduced.
3. The Heston model sampling method has been developed.
4. In addition, to check the practicality of the method used in Point 2, convergence of GARCH (1, 1) model has been tested for stationary and Gamma distribution, taking into account the correlation coefficient.
5. Using the autocorrelation correction model (Point 2), the Black-Scholes option-pricing model and the Option Greeks associated within it have been transformed. This formula allows for more accurate pricing, given the heavily tail asset yields. A summary of the autocorrelation method has been considered, based on stock option techniques of Tesla Motors Inc. and with Monte Carlo's simulation of the expected volatility and option pricing. The defined system helps to create its own view of the financial market situation, option prices and make a reasoned decision regarding the buy/sell decision of shares or options.

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