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OPTIMIZATION OF AIRCRAFT TECHNICAL OPERATION PARAMETERS TO IMPROVE FLIGHT REGULARITY

Doctoral Thesis

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CONTENT

ABSTRACT

This study focuses on improving the operational management of aircraft maintenance processes to enhance flight regularity. The relevance of the topic stems from increasing demands for reliability and punctuality in air transportation, especially under conditions of limited resources and intensive fleet operation. The object of the research is the activity of the airline RAF-AVIA, which operates a regional network of flights with various types of airports and maintenance forms.

In Chapter 1, an analysis of the causes of flight delays related to aircraft technical condition is presented, along with a review of the specifics of operational maintenance management, airport classification, and the key factors affecting flight regularity. Additionally, public data was used to analyze punctuality statistics.

Chapter 2 presents a mathematical model for delay formation based on the dynamics of aircraft technical states and probabilistic characteristics of recovery processes. The model accounts for state transition probabilities, flight network structure, and specific airport parameters. The developed application tool, implementing the proposed mathematical models, enables the calculation of key flight regularity indicators and allows evaluation of the influence of various operational parameters.

Chapter 3 includes an in-depth analysis of aircraft state dynamics at both base and non-based airports, proposes a method for evaluating the significance of operational factors on departure delay durations, and presents practical recommendations for RAF-AVIA aimed at optimizing maintenance processes and reducing the expected duration of flight delays.

Chapter 4 develops an objective-function-based optimization framework for AOG-critical defect scenarios, linking probabilistic state dynamics with recovery and logistics parameters to evaluate and select delay-mitigation strategies at the route level.

This work is aimed at improving the efficiency of aircraft maintenance management through modeling, quantitative analysis, and the application of evidence-based decisions tailored to the operational context of a specific airline.

The application tool operates solely on airline-provided input parameters and accumulated operational statistics, ensuring that all calculations are based on real empirical data.

INTRODUCTION

Modern airlines operate under strict safety constraints and increasing pressure to maintain high levels of schedule reliability. In regional airline networks, technical disruptions often propagate across multiple routes, creating cumulative delays and operational instability. Despite the importance of operational aircraft maintenance management, quantitative tools for evaluating recovery processes and predicting technical delay formation remain limited.

This dissertation develops a probabilistic model of aircraft technical state evolution and recovery processes within an airline route network. The model integrates defect occurrence probability, MEL-based operational constraints, airport technical capability, and recovery time parameters. The proposed approach enables quantitative assessment of technical delay formation and supports data-driven operational decision-making. The model is implemented in the FlightSync application tool and validated using real operational data from the Latvian airline RAF-AVIA.

Research Relevance

The relevance of this research is driven by the need to improve flight regularity through quantitative optimization of operational aircraft maintenance within airline networks:

- Flight regularity depends on how effectively aircraft maintenance and technical operations are managed;
- In an airline network, technical delays are caused not only by failures and technical malfunctions, but also by organizational factors such as recovery time, resource logistics, staff availability, and spare parts support;
- Airlines differ in fleet structure, resources, and maintenance organization; therefore, standardized solutions cannot ensure stable performance;
- Improving flight regularity requires a systematic approach based on mathematical modeling and optimization of aircraft technical operation parameters considering resource availability.

Research objectives

Improving flight regularity through optimization of operational aircraft maintenance parameters based on mathematical models of recovery processes within the airline route network.

To achieve this goal, the following research tasks were defined:

1. To analyze the current state of the flight regularity problem in civil aviation and its impact on airline network operations;
2. To analyze aircraft failures and technical malfunctions during operation, assess recovery capability within the airline network, and determine operational factors influencing technical delay probability;
3. To develop mathematical models of aircraft state assessment and recovery processes in accordance with MEL (Minimum Equipment List) requirements within airline network airports, aimed at reducing both the probability and duration of departure delays;
4. To develop application tool for implementing the proposed mathematical models within operational maintenance management;
5. To test and validate the developed application tool using operational and statistical data from the RAF-AVIA airline.

Methods of the research

The following methods were applied:

1. Methods of probability theory for the development of mathematical models;
2. Matrix method of analysis and modeling;
3. Statistical method;

4. Method of aircraft technical condition identification.

Scientific novelty of the research

The scientific novelty of this dissertation is reflected in the following contributions:

1. Developed graphical models of aircraft states and state transitions within an airline network, including both base and outstation airports, in the framework of aircraft technical operations;
2. Proposed mathematical models for aircraft state assessment in accordance with Minimum Equipment List (MEL) requirements for airline network airports in the event of in-flight failures and subsequent recovery, aimed at minimizing both the probability and duration of departure delays and improving flight regularity;
3. Developed and implemented an application tool, FlightSync, based on the proposed mathematical models and cumulative technical delay calculation algorithms. The application tool enables analysis of key factors affecting flight regularity and improves the efficiency of aircraft technical operations within the airline network. Testing on real operational data from RAF-AVIA confirmed the practical applicability and effectiveness of the solution.

Practical significance

The practical use of the developed models in airline operations significantly reduces both the probability and average duration of flight delays within the airline network in the event of technical failures. This is achieved through optimization of recovery processes, ensuring efficient allocation of maintenance resources, and forecasting of technical delays within the aircraft technical operation.

Theses to be defended

1. Graphical models of aircraft states and state transitions during failures and technical defects, for both base and outstation airports within the airline network in the aircraft technical operations.
2. Mathematical models for aircraft state assessment in accordance with MEL (Minimum Equipment List) requirements within airline network airports during failures and technical defects, as well as recovery models aimed at reducing both the probability and duration of departure delays and improving flight regularity.
3. Application tool that implements the developed mathematical models and provides calculation and forecasting of expected technical delays at airports within the airline network.

Results of the work

1. A comprehensive analysis of the current state of the flight regularity problem in civil aviation and its impact on airline network operations was conducted. The analysis examined existing approaches to flight regularity management and clarified the role of operational aircraft maintenance in maintaining stable performance within an airline network;

2. Aircraft failures and technical malfunctions during operation were analyzed using operational data, recovery capability within the airline network was assessed, and operational factors influencing technical delay probability were determined. The study established the relationship between defect occurrence, recovery conditions at network airports, and delay formation within the airline network;
3. Mathematical models of aircraft state assessment and recovery processes in accordance with MEL (Minimum Equipment List) requirements within airline network airports were developed. The models describe aircraft state transitions under failure conditions and defined recovery scenarios, and are aimed at reducing both the probability and duration of departure delays;
4. The FlightSync application tool was developed to implement the proposed probabilistic mathematical models within operational aircraft maintenance management. The application enables calculation of cumulative technical delays and practical application of the developed models within airline operations;
5. The FlightSync application tool was tested and validated using operational and statistical data from the RAF-AVIA airline. The validation confirmed the correctness of the implemented probabilistic models and their applicability in supporting operational aircraft maintenance management within the airline network.

Accuracy of research results

All research results are based on practical developments carried out by the author, including regulatory, technical, and organizational documentation, as well as actual operational data and interaction with airports within the RAF-AVIA airline network.

Thesis approbation

The research findings have been published in 6 scientific papers and presented at 9 international scientific conferences held in Latvia and Poland, including a SCOPUS-indexed publication.

Publications

1. Recovery Model for Aircraft at Airports Within the Airline Network of the Airline Company (2025). A. Zabirov, V. Šestakovs, J. Maklakovs, N. Krontals, I. Blumbergs. International Review of Aerospace Engineering (IREASE) - SCOPUS. DOI: https://doi.org/10.15866/irease.v18i1.25566;
2. Operational Management of Technological Processes for Preparing Aircraft for Departure at the Airport as a Multi-Criteria Task (2024). A. Zabirov, J. Soldatova, Z. Zabirov, V. Šestakovs. Latvian Journal of Physics and Technical Sciences - SCOPUS. DOI: https://doi.org/10.2478/lpts-2024-0047;
3. Solving Technological Problems of Flight Services at Airport Involving Composite Materials (2024). M. Romele, A. Zabirov, R. Chatys, V. Šestakovs. Composites Theory and Practice - SCOPUS. DOI: https://doi.org/10.62753/ctp.2024.07.2.2;

4. Modelling a Quantitative Assessment Method of Pilots' Performance in Evidence-Based Training (2022). Z. Zabirov, A. Zabirov, V. Šestakovs. Aviation - SCOPUS. DOI: https://doi.org/10.3846/aviation.2022.18037;
5. Analysis of Approaches to Assessing Flight Delays Due to Technical Issues at Airline Network Airports Within the Operational Management Framework (2024). A. Zabirov, Z. Zabirov, V. Shestakov. DOI: https://doi.org/10.2478/tar-2024-0008;
6. Analysis of the Strapdown Inertial Navigation System (SINS) Error Genesis (2018). A. Zabirov, P. Trifonovs-Bogdanovs. Transport and Aerospace Engineering. DOI: https://doi.org/10.1109/MAES.2005.1499250;

International Scientific Conferences

1. Development of Theoretical and Methodological Assessment Approaches of Airline Safety Culture as a Risk Factor (2023). A. Zabirov, J. Maklakovs, V. Šestakovs, N. Kuleshov. Instytut Lotnictwa Warszawa - SEMINARIUM V;
2. Ensuring Flight Safety Based on Improved Accuracy Rnav (Regional Navigation) Performance-Based Navigation Systems in Airspace Republic of Uzbekistan (2023). A. Zabirov, Z. Zabirov, V. Šestakovs. Instytut Lotnictwa Warszawa - SEMINARIUM V;
3. An Approach to Improving the Accuracy of Determining the Parameters of a Strap Down Inertial Navigation System (2022). A. Zabirov, Z. Zabirov, V. Šestakovs. Instytut Lotnictwa Warszawa - SEMINARIUM IV;
4. Methodological Approach of Management by a Human Factor in the System of Technical Exploitation (2022). A. Suharev, K. Stanisław Szafran, V. Šestakovs, A. Zabirov. Instytut Lotnictwa Warszawa - SEMINARIUM IV;
5. Error Formation in a Human Operator Decision-Making Process in Flight (2020). A. Zabirov, Z. Zabirov, V. Šestakovs. Transport Means;
6. Modelling a Quantitative Assessment Method of Pilots' Performance in Evidence-Based Training (2020). A. Zabirov, Z. Zabirov, V. Šestakovs. Riga Aviation Forum;
7. Analysis Of The Strapdown Inertial Navigation System Structure (2020). A. Zabirov, Z. Zabirov, V. Šestakovs, Pjotrs Trifonovs-Bogdanovs, T. Rogalski. READ – Research & Education in Aircraft Design;
8. Optimization of the Strapdown Inertial Navigation System Structure of Aircrafts (2020). A. Zabirov, Z. Zabirov, V. Šestakovs. 61ST INTERNATIONAL SCIENTIFIC CONFERENCE OF RTU.
9. Error Analysis of the Modern Aircraft Navigation Systems (2019). A. Urbahs, P. Trifonovs-Bogdanovs, A. Zabirov, K. Mamay. The 2nd Aviation and Space Congress.

List of abbreviations

- MCC – Maintenance Control Center
- CAMO – Continuing Airworthiness Management Organization
- OCC – Operations Control Centre
- MEL – Minimum Equipment List
- AOG – Aircraft on Ground

- MPD – Maintenance Planning Document
- AMP – Approved Maintenance Program
- IATA – International Air Transport Association
- ICAO – International Civil Aviation Organization
- EASA – European Union Aviation Safety Agency
- FAA – Federal Aviation Administration
- A/C – Aircraft
- TO – Technical Operation

Structure of the work

The work contains an abstract, introduction, 4 chapters, summary, conclusion, bibliography, appendices, 42 figures, 7 tables, 146 pages, and 75 references.

Work includes the following main parts:

1. Abstract. Summarizes the purpose of the research: to improve flight regularity by enhancing operational management of aircraft maintenance. Highlights the use of probabilistic modeling and application tool implementation based on RAF-AVIA operations;
2. Introduction. Explains the relevance of flight delays due to technical issues and defines the research goal, objectives, methods, and the practical significance of the study for regional airline operations;
3. Chapter 1. Analyzes causes of delays related to aircraft technical condition and operational management. Describes airport classifications, airline network structure, and introduces the concept of technical state transitions;
4. Chapter 2. Presents a probabilistic model of aircraft state degradation and restoration. Introduces dual-level MEL logic, recovery/delivery time modeling, and integrates these into an application tool (FlightSync) for scenario analysis;
5. Chapter 3. Applies the model to RAF-AVIA operations. Identifies key operational factors affecting delays and provides prioritized recommendations for improving maintenance processes based on simulation results;
6. Chapter 4. Formulates an objective-function-based approach to optimizing operational aircraft maintenance management. The chapter introduces a quantitative criterion representing the expected total technical delay of a route, analyzes the influence of managerial measures through adjustments of probabilistic and temporal model parameters, and demonstrates the effectiveness of combined strategies using a detailed route-based case study for the RAF-AVIA airline network;
7. Conclusion. Confirms the research objectives were achieved. The developed model and application tool support data-driven decision-making to reduce technical delays and improve flight regularity;
8. References. A structured list of 67 sources, including ICAO, EASA, FAA, and academic publications on reliability, maintenance, and airline operations;
9. Appendices. Program code of FlightSync application tool. AOG Statistics Data.

1. ANALYSIS OF FACTORS AFFECTING FLIGHT REGULARITY AND THE EFFICIENCY OF OPERATIONAL MANAGEMENT IN AIRCRAFT MAINTENANCE

Introduction

Flight regularity serves as a fundamental metric for assessing how effectively an airline delivers scheduled air services. Delays not only interfere with the flow of subsequent operations but can also disrupt coordination among internal units and, at times, partner entities in the civil aviation sector. From the passenger's perspective, both the frequency and length of delays matter significantly. On short and medium-distance routes, prolonged delays may prompt travelers to consider alternative modes of transport. Conversely, on long-haul routes where alternatives are typically unavailable, passengers are compelled to wait – shifting the burden of delay primarily onto the airline in the form of financial losses, which escalate with the duration of the disruption.

To improve departure punctuality and reduce taxi-out delays, many airports have adopted collaborative decision-making systems (A-CDM), which enable stakeholders such as air traffic control, ground handling, and airlines to share real-time operational data [50]. This approach ensures real-time information sharing between airlines, ground handlers, and air traffic control, allowing better turnaround management and slot usage at congested airports. The Airport CDM Implementation Manual outlines standardized procedures to support efficient pre-departure sequencing and reduce avoidable delay propagation [1].

Globally, the process of tracking and disseminating flight punctuality metrics has been largely delegated to automated systems, which interface directly with aviation databases. These systems are typically maintained by national authorities or civil aviation institutions and operate without manual intervention. The type and complexity of the data retrieved from basic delay counts to detailed performance breakdowns are tailored to match the evolving analytical or regulatory requirements of different user groups.

Airline associations, such as IATA (International Air Transport Association), regularly report the performance of their member airlines, including flight regularity metrics. Evaluations are conducted based on both departure and arrival punctuality indicators.

Improving on-time performance requires a cross-functional approach involving operations control, airport coordination, ground handling efficiency, and real-time data sharing. Airlines that adopt integrated delay management practices and align accountability across departments typically achieve significantly higher punctuality scores [2].

To facilitate consistent delay reporting across the industry, the International Air Transport Association (IATA) has established a unified classification system for identifying the causes of delays. These codes are grouped into ten major categories, each reflecting a specific area of operational responsibility, as outlined in Table 1.1 [3].

Table 1.1

Percentage distribution of all departure delay causes by IATA categories in 2025 [3]

IATA Code	Definition
0-9	Others & Airline internal codes
11-18	Passenger and baggage handling
21-29	Cargo and mail
31-39	Aircraft and ramp handling
41-48	Technical and aircraft equipment
51-58	Damage to aircraft and automated equipment failure
61-69	Flight operations and crewing
71-77	Weather
81-89	Air traffic flow management / Airport and Governmental Authorities
91-96	Reactionary delay
97-99	Miscellaneous

Disruption management is a key operational discipline aimed at minimizing the negative impact of unforeseen events such as delays, cancellations, or equipment unavailability. Effective strategies in disruption management help airlines recover schedules, allocate resources efficiently, and maintain service continuity during irregular operations [4]. To address uncertainties in maintenance scheduling, a rolling horizon framework can be applied, enabling dynamic re-optimization of maintenance tasks based on updated operational data [55]. To improve the accuracy of turnaround planning, predictive models can estimate required maintenance hours based on specific MEL-related scenarios and historical repair data [56].

Within the European air transport system, flight regularity is assessed separately for airports and airline operators. Airports are primarily evaluated based on the punctuality of departures, whereas airlines are judged by the accuracy of arrival times. This distinction reflects the practical reality that passengers are more concerned with reaching their destination on time than with leaving precisely as scheduled.

Airlines, whether operating independently or through alliances, frequently assess airport efficiency by tracking departure timing. The underlying goal is to hold airport operators accountable for delay prevention, foster coordination between ground services and carriers, and reduce turnaround inefficiencies. At airports with high operational discipline, on-time departure rates can exceed 90%.

Delay statistics are compiled and processed through centralized air traffic management systems to analyze delay causes and trends [49]. EUROCONTROL's regulatory framework outlines the procedures for managing flow and capacity within European airspace [6]. Through the Central Flow Management Unit (CFMU), the system maintains comprehensive records of filed flight plans and actual flight paths, making it possible to monitor delays by both airport and route under the ATFM environment.

Effective delay management in air traffic flow systems requires coordinated actions between the network manager, air navigation service providers, and airport stakeholders. Tools such as dynamic slot allocation, regulation planning, and collaborative decision-making (CDM) help mitigate enroute and arrival delays while maintaining overall network efficiency [7].

According to the Association of European Airlines, delays are defined as exceeding 15 minutes, with punctuality measured by departures for airports and arrivals for airlines. The statistics are categorized by flight distance to account for greater wind sensitivity on long-haul routes.

Performance metrics across European airlines often reveal disparities between long-haul and regional operations. For instance, Lufthansa has, in certain years, reported perfect regularity on intercontinental routes, while achieving lower rankings such as 13th place in European categories, despite completing 98.9% of scheduled flights.

Punctuality is evaluated by delay brackets, typically as follows:

- 15–30 minutes (minor delays, high punctuality);
- 31–120 minutes (medium punctuality);
- 121–360 minutes (low punctuality);
- More than 360 minutes (risk zone).

Flights departing within 15 minutes of schedule are generally not considered delayed.

Maintaining consistent flight regularity imposes notable operational costs, including:

- maintaining a reserve fleet of standby aircraft;
- investing in newer, more reliable aircraft;
- reduced peak-hour flight density and associated revenue losses;
- limitations on aircraft utilization;
- and organizational restructuring aimed at streamlining processes.

Aircraft reliability is a fundamental factor that influences flight regularity by determining the likelihood of technical delays and unscheduled maintenance events. Accurate assessment of reliability indicators enables data-driven planning of inspections and interventions to ensure the uninterrupted operation of aircraft systems [8].

Flight delays directly undermine airline profitability and weaken competitive standing in the market. Industry estimates from 2010 indicate that the cost of recovering disrupted schedules across Europe reached €1.25 billion translating to an average financial impact of €81 for every minute of delay.

In engineering systems such as airline operations, the analysis of risk is crucial to identifying events with high probability and severe impact. Integrating probabilistic assessment into operational planning enables decision-makers to prioritize mitigation strategies and improve overall system robustness under uncertainty [9]. This highlights the need for a structured approach to evaluating and mitigating the operational risks associated with delay events [62].

According to specialists from the USA, in 2007 the cost of flight delays for the country’s economy amounted to 32.9 billion dollars, of which 8.3 billion were borne by airlines due to increased operational costs [10].

Analysis shows that methods and models for assessing airline flight delays are underdeveloped, leading to an inability to accurately evaluate delays and to justify the development of measures to reduce them. The mathematical apparatus typically used in studying flight delays mostly involves analyzing distributions of certain delay cause parameters using various approximations, etc. Therefore, improving these methods is a current and relevant problem.

The present study addresses these challenges by proposing mathematical modeling approaches to optimize the operational control of aircraft maintenance, with the ultimate objective of enhancing flight regularity.

According to ICAO’s 2023 Safety Report, enhancing operational safety and reducing accident rates remain top priorities for global aviation stakeholders. The report outlines key risk categories and promotes the use of safety performance indicators (SPIs) to monitor and manage safety outcomes at both national and organizational levels. These insights support the ongoing development of safety-focused operational models in aviation [11].

Standardized technical documentation plays a key role in the efficiency and consistency of aircraft maintenance. According to ATA Spec 100 and iSpec 2200, technical manuals are structured to provide uniform formats for maintenance tasks, component information, and troubleshooting procedures. This ensures that technicians across organizations interpret and perform required tasks consistently, contributing to higher maintenance quality and improved operational regularity [12].

1.1. Analysis of the state of flight regularity in civil aviation

Contributing to higher flight regularity is a key indicator of performance for any airline, directly influencing passenger satisfaction, operational efficiency, and financial sustainability. Inconsistent flight schedules result not only in inconvenience for travelers, but also in cascading disruptions across airline networks, missed connections, crew displacement, and increased operational costs.

From a regulatory perspective, regularity is closely monitored by aviation authorities and integrated into safety oversight and performance evaluations. Thus, a systematic understanding of delay formation and its root causes is critical to developing effective management models aimed at improving reliability and minimizing risks.

The issue of flight regularity remains one of the most pressing challenges in civil aviation, as delays continue to disrupt schedules and reduce overall system efficiency. Understanding the root causes of such delays is essential for improving operational resilience across the industry.

Flight delays may stem from both controllable and uncontrollable factors, the most frequent of which include the following:

- unfavorable meteorological conditions;
- delayed passenger arrivals;
- urgent medical needs of individuals on board;
- late scheduling of commercial flights;
- air traffic control restrictions;
- errors or inefficiencies involving airport staff, ground handlers, or flight crews;
- security-related procedures;
- technical inspections exceeding expected duration;
- technical malfunctions of the aircraft itself, and others.

Among these, the most significant contributor accounting for more than 50% of disruptions is the reduced operational capability of aircraft in poor weather conditions. The remaining factors are largely tied to organizational inefficiencies within airline and airport management systems [13].

It is important to note that delay attribution methodologies differ internationally. Comparative research reveals inconsistencies between American and European data.

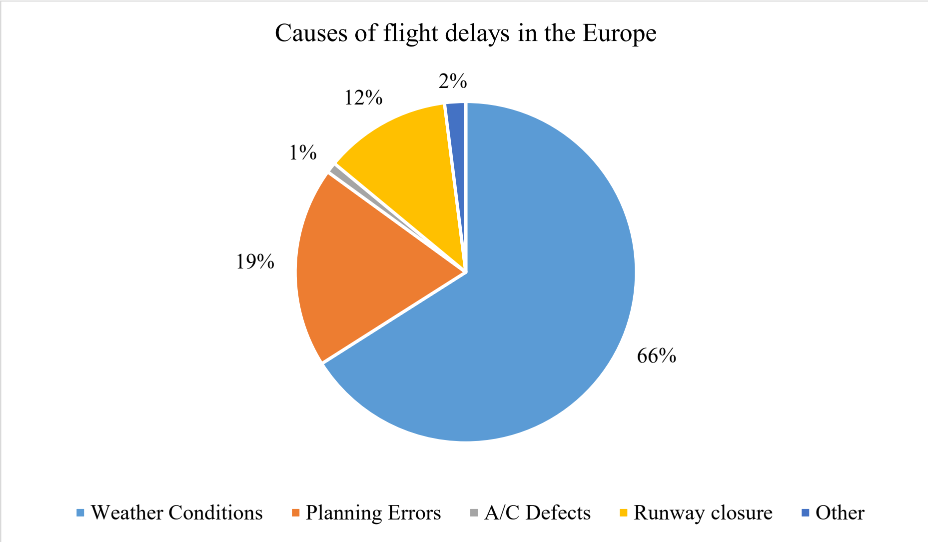
European analyses often identify late aircraft arrivals causing knock-on effects across flight schedules as the leading source of major delays. Conversely, U.S.-based studies frequently cite weather-related factors as the primary cause, especially in the context of already saturated airport infrastructure. In addition, insufficient planning that ignores actual airport capacity is considered a critical operational weakness contributing to delay propagation.

A breakdown of delay causes and their average durations for the year 2017 is provided in Table 1.2 and visualized in Figure 1.1 [14].

Table 1.2

Causes and duration of delays (%) in 2017 (Europe) [14]

Cause of delay / Duration	**< 15 min**	**15–30 min**	**31–60 min**	**1:01–2:00 hrs**	**> 2:00 hrs**
Uncategorized delays (various causes) [%]	32,61	5,98	2,99	2,77	2,38
Airline operational causes [%]	2,20	4,36	4,76	4,76	2,79
Passenger and baggage-related delays [%]	2,97	3,46	1,68	0,81	0,55
Delays caused by aircraft servicing by suppliers (loading, refueling, catering) [%]	2,44	2,83	0,95	0,53	0,20
Delays caused by aircraft maintenance or defects [%]	1,85	3,98	5,08	6,72	12,83
Delays due to operational control and crew duty regulations [%]	3,24	4,43	2,96	2,46	2,00
Delays caused by air traffic management [%]	12,76	15,24	8,58	4,60	2,50
Delays caused by airport restrictions [%]	10,81	6,09	2,06	1,02	0,67
Delays caused by previous flight delay (reactionary delay) [%]	31,12	53,64	70,94	76,41	76,08



1.1. fig. Causes of Flight Delays in Europe [14].

Comprehensive statistical reviews have shown that a considerable share of delays stems from the technical unavailability of aircraft at the time of scheduled departure. These issues fall under the responsibility of the airline's maintenance and engineering division [60] and may involve:

- identification of aircraft defects;
- replacement of the aircraft with an alternative unit;
- unplanned or extended technical inspections;
- and failure to complete flight preparations due to maintenance-related delays or personnel-related issues.

At Lufthansa, during the period from January 1, 2020, to December 31, 2020, the number of flight departure delays caused by:

- flight crews – approximately 1,200 delays;
- ground services – approximately 900 delays;
- technical services – approximately 3,500 delays.

In the summer of 2024, Swiss International Air Lines faced significant punctuality issues, with nearly 14,000 flights delayed out of approximately 32,222 scheduled, resulting in an on-time performance rate of just 57% placing SWISS among the bottom three airlines in European punctuality rankings, alongside ITA Airways and EasyJet [15].

The most common factors contributing to these delays were:

- severe weather events such as thunderstorms and high winds affecting Zurich and Geneva;
- bottlenecks in Europe's air traffic control system leading to knock-on delays;
- disruptive restrictions at Zurich Airport, including night flight bans and intersecting runway configurations;

- technical malfunctions within the ATM system (e.g. Skyguide software failure causing widespread delays);
- crew shortages and labour-related bottlenecks, exacerbated by new regulations and retraining efforts.

Reliability-Centered Maintenance (RCM) is a structured framework used to identify failure modes, analyze their consequences, and develop maintenance strategies focused on preserving system functionality. This approach allows operators to prioritize maintenance tasks based on risk and criticality, ensuring safety while optimizing resource use [16].

The mathematical apparatus used in the study of delays is based on methods for analyzing distributions of certain parameters, investigating dependencies between parameters using various types of approximations, as well as graphical solutions to problems involving joint description of different indicators under specific constraints.

The study of flight regularity and delay behavior involves the application of various statistical methods and classification criteria. The most relevant types of analysis include:

- time series analysis of aircraft departure and arrival regularity;
- evaluation of daily delay frequency distributions;
- distribution analysis of delay durations;
- categorization of delays by cause codes;
- time series analysis of delays by individual codes or sources;
- comparison of average delay durations across different aircraft types;
- examination of the dynamics in departure regularity over time;
- analysis of delay patterns by flight number or route;
- evaluation of delay distributions across different time segments of the day;
- and investigation of regularity trends and delay frequency across days of the week and months of the year.

Based on this, analysis of the average delay time distribution shows the presence of pronounced peaks [17], which can be explained by scheduling policies, turnaround constraints, or external events.

As experience in airline and airport operations shows, delay propagation is often associated with procedural bottlenecks and poor coordination during turnaround [52]. Under these conditions, any recommendations given to airline or airport management regarding the implementation of organizational measures to reduce potential profit losses can only be of a predictive and probabilistic nature. This requires the development of appropriate approaches and mathematical models.

Analysis of flight departure regularity statistics at airports indicates that one of the causes of flight delays and disruptions in regularity is the insufficient efficiency of the operational management system for flight servicing and aircraft preparation for departure.

Therefore, the aim of this study is to address the problem at the level of operational management of an airline’s aircraft fleet within the functioning network of air routes.

1.2. Operational Planning and Control in Airline Service Systems

The efficiency of airline operations heavily depends on how well short-term planning and real-time control processes are implemented. The airline's management framework typically consists of two interrelated levels:

- the strategic level, which defines the guiding principles and long-term decision-making criteria for enterprise development;
- and the operational level, focused on the execution of daily tasks required for stable performance and flight regularity.

Operational control plays a decisive role in managing core services such as aircraft ground handling, commercial logistics, technical inspections, and dispatch coordination. One of the most critical stages is pre-departure preparation, which must account for complex interdependencies between aircraft readiness, crew scheduling, ground resources, and regulatory limitations [18], [19].

Accurate resource allocation is a central task at this level: from aircraft and personnel to fuel supplies, tools, and transport units. Any miscalculation can result additional costs. Operational readiness also depends on strict compliance with procedural and safety standards, which are detailed in internationally recognized documentation, including FAA handbooks and maintenance protocols [20].

The Daily Flight Plan (DFP) is a core tool in airline service coordination. It defines all scheduled arrivals, departures, and transit operations at the airport. The implementation of the DFP requires synchronization across airport departments and availability of essential servicing facilities (SF).

Real-time monitoring is essential within the operational loop. When deviations from schedule parameters occur, a feedback mechanism is activated to correct operations and restore the system to its planned path.

From a systems engineering perspective, operational control is based on a mathematical model describing aircraft processes as technological chains composed of interconnected functional elements.

Integration of flight planning, technical servicing, and execution activities contributes to cost efficiency. This reduces fuel consumption, enhances aircraft utilization, and ensures compliance with airworthiness standards [21].

EU-based operators are subject to EASA operational regulations, including CAT.GEN.MPA and ORO.GEN, which set legal frameworks for flight safety, crew performance, and operational accountability [22].

Finally, airline production-dispatch units are responsible for coordinating service department activities and ensuring timely communication with passengers, service providers, and airport stakeholders.

1.3. Departure delays in the aircraft preparation cycle

Departure delays frequently originate within the aircraft preparation cycle, particularly during the execution of tasks governed by the airline's operational control loop. This stage involves two major technical domains:

- maintenance operations performed by the airline's internal engineering and technical units;
- pre-flight preparation procedures conducted by airport ground services.

Both areas represent structured technological processes composed of individual operations. Each operation must adhere to established regulatory procedures and is associated with specific time and resource requirements. Deviation from these norms often leads to schedule disruption and negatively impacts overall flight regularity.

In recent years, the aviation industry has increasingly adopted lean maintenance principles aimed at reducing inefficiencies and aligning technical procedures with real-time operational needs. As noted by Dombrowski and Zahn, this approach emphasizes minimizing waste, streamlining workflows, and enhancing resource utilization contributing directly to reduced turnaround times [23].

Regulatory oversight further reinforces compliance and performance monitoring. In the United States, the FAA's Order 8900.1 outlines detailed protocols for evaluating maintenance execution, identifying deviations, and implementing corrective actions during technical audits [24].

Given the complex and time-sensitive nature of these processes, this study will focus specifically on the operational management loop of aircraft technical operations, examining how internal factors within this system contribute to delays at airports across the airline's route network.

In practice, even minor inefficiencies in task coordination such as delayed start of fueling, tool unavailability, or miscommunication between line maintenance and dispatch units can propagate across the operational cycle, resulting in cumulative delays. These seemingly small disruptions highlight the importance of not only optimizing individual procedures but also managing interdependencies across departments. Consequently, improving internal workflows and information exchange within the operational management loop becomes essential for enhancing flight regularity and minimizing downtime.

These disruptions often go unnoticed in traditional performance reports but have a compounding effect on system-wide punctuality. Addressing such inefficiencies requires not only procedural standardization but also a shift toward proactive, data-driven decision-making within maintenance and ground operations.

1.4. Route-Based Operational Control and Aircraft Technical States

The efficient management of aircraft within an airline's route network requires continuous synchronization between flight scheduling and maintenance availability. To capture the complexity of this interaction, the aircraft's technical operation is typically divided into two core phases:

- ground-based maintenance procedures, including inspections, repairs, and turnaround servicing;
- and in-flight technical operation, which reflects the aircraft's performance and systems behavior during active flight phases.

Each phase consists of a finite set of operational states. These states, along with the transitions between them, can be formally represented as a directed graph, where vertices correspond to specific stages of the operation, and edges represent the dependencies or flow of execution.

In route-based planning, the aircraft is assigned to a sequence of flights between airports. To describe its position in the chain, a discrete temporal parameter n is introduced, representing the index of each flight within a rotation. For a total of N flights, this parameter takes values $n \in \{1, 2, ..., N\}$.

Between successive flights, the aircraft undergoes a turnaround period. If no recovery or technical intervention is required, this period is short and consistent. However, when unscheduled maintenance or fault rectification is needed, recovery time must be added, directly affecting the total time between flights and potentially leading to delays.

Airports along the route are classified by their technical capabilities into three categories:

- B – Base Airport (equipped for full maintenance and turnaround);
- T – Transit Airport (limited/no maintenance capability);
- TB – Transit Base (partial maintenance capacity).

This classification is essential when modeling system reliability and predicting delays, as it directly influences where and how an aircraft can be recovered in case of malfunction.

Each flight route is represented as a sequence of flights with turnarounds at airports of different types. The structure of flight routes in the RAF-AVIA airline network is presented in Table 1.3.

Table 1.3

Structure of the flight network of the RAF-AVIA airline [66]

№	Flight Structure (Graphical)	Flight Parameters
1	*B – T – B*	1/2
2	*B – T – T – B*	1/3
3	*B – T – TB – B*	1/3
4	*B – T – T – T – B*	1/4
5	*B – T – T – TB – B*	1/4
6	*B – T – TB – T – B*	1/4
7	*B – TB – TB – T – TB – B*	1/5
8	*B – TB – B – TB – TB – B*	1/5
9	*B – TB – T – TB – T – TB – B*	1/6
10	*B – T – T – T – TB – T – TB – B*	1/7
11	*B – TB – T – TB – T – T – TB – B*	1/7
12	*B – TB – T – T – TB – T – TB – B*	1/7

The distinguishing characteristics of airports are presented in Table 1.4.

Table 1.4

Characteristics of Airports in the Airline Network [66]

№	Characteristics	Airport Type			Notes
		T	*TB*	*B*	
1	Probability of availability of spare parts necessary for aircraft recovery	P_{SP}^{T}	P_{SP}^{TB}	P_{SP}^{B}	-
2	Probabilistic characteristics of aircraft recovery time (T_R)	$F^T(t)$	$F^{TB}(t)$	$F^B(t)$	May be specified using alternative methods
3	Probabilistic characteristics of operational delivery time of necessary spare parts if unavailable at the airport (T_D)	$G^T(t)$	$G^{TB}(t)$	$G^B(t)$	May be specified using alternative methods
4	Probability of aircraft loop-back on the next flight at the base airport	-	-	P_{LB}	-
5	Probability of availability of a reserve aircraft at the base airport for flight execution	-	-	P_{RES}	-
6	Availability of specialists for aircraft recovery work	No	Yes	Yes	-
7	Average aircraft turnaround time at the airport	T_{AT}^{T}	T_{AT}^{TB}	T_{AT}^{B}	For type “B” in case the aircraft is loop-back into the next flight

Within the framework of the operational management loop of the airline’s aircraft fleet technical operation, the technical operation process can be represented as a sequence of flight routes, which in turn consist of a sequence of flights between airports in the airline’s route network over a calendar time interval during which the aircraft remains within this management loop.

The aircraft technical operation cycle can be effectively visualized using a graph-based structure, as illustrated in Figure 1.2. In this representation, each node denotes a specific operational or maintenance state, while the edges define the sequence and dependency of transitions between those states.

The aircraft’s progression through the various stages of this process is determined by several key factors:

- operational demand, dictated by the airline’s flight schedule and the aircraft’s assigned rotation within the route network;
- off-cycle maintenance requirements, such as scheduled overhauls or unscheduled repairs, occurring outside the regular operational loop;
- recovery needs arising at en-route airports due to in-flight deterioration or technical faults;
- the maintenance and recovery capabilities available at individual airports within the airline’s operational system;
- and the set of allowable malfunctions defined by the aircraft’s design limitations and operational tolerance levels.

Within the framework of structural airframe maintenance, regulatory resources such as the FAA’s Airframe Handbook provide detailed guidance on inspecting and servicing critical fuselage elements including skin panels, access doors, fairings, and drainage systems. Ensuring the integrity of these components is vital for flight safety and represents a fundamental task within periodic maintenance activities [25].

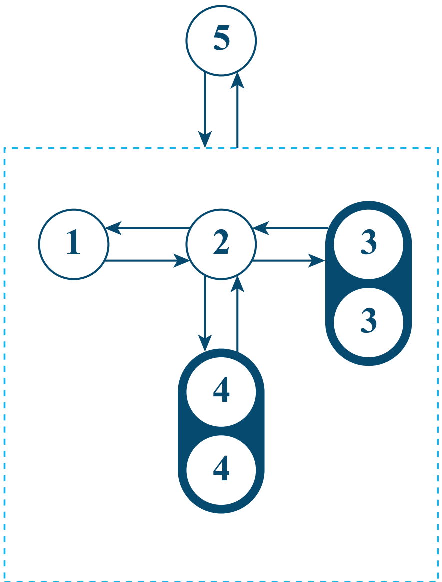
1.5. Aircraft states and transitions across the stages of its utilization process at the operational management loop level

The technical operation of an aircraft within the airline's operational control system can be segmented into distinct stages corresponding to its physical and functional status across the route network. These stages include:

1. presence at the base airport;
2. active in-flight operation;
3. turnaround at a transit airport;
4. stay at a transit base with limited maintenance capacity;
5. placement outside the operational control loop for scheduled or unscheduled maintenance.

The evolution of the aircraft's state through these stages can be modeled as a Markov chain, where transitions occur with defined conditional probabilities based on the current operational context and system constraints [59].

The state-transition graph of the aircraft across the stages of the technical operation process at the operational management loop level is presented in Figure 1.2 (structure adapted by the author based on operational models in [18] and [37]).



1.2. fig. State-Transition Graph of the Aircraft Across the Stages of the Technical Operation Process (at the Operational Management Loop Level) [66].

Let us examine in more detail the content and structure of the stages of the aircraft technical operation system in accordance with the sequence of transitions and states of the aircraft at the operational management loop level.

1.6. Aircraft maintenance on the ground during pre-flight preparation

Pre-flight preparation of all aircraft necessarily includes a set of ground maintenance procedures aimed at ensuring airworthiness and readiness for departure. To systematically examine the interrelation between individual technological steps within this process, and to extract the quantitative parameters required for constructing a mathematical framework for operational planning, the use of graph-theoretic models is proposed. These models offer a structured representation of the sequence and dependency of operations, enabling formal analysis and optimization of the pre-flight workflow.

According to Lee et al., effective logistics and operations management in aviation relies on coordinated planning of technical support, inventory availability, and time-sensitive maintenance scheduling. Optimized logistics flows reduce the risk of ground time overruns and ensure seamless integration of aircraft into the operational cycle [26].

Graphical models make it possible to identify quantitative relationships between graph nodes for improving aircraft technical operation processes using statistical research methods.

The aircraft state graph will be examined using the example of a network-based process flow chart for pre-flight preparation in the most general case – a transit flight, see Figure 1.3.

1.7. Process flow chart of comprehensive preparation of the aircraft for departure at a transit airport

All aircraft undergo ground handling during preparation for a flight at a transit airport. The process flow chart of comprehensive preparation includes the following tasks: aircraft reception, pre-flight preparation, and dispatch for flight. These procedures follow the general maintenance flow described in FAA guidance materials [24].

Aircraft reception involves guidance assistance using visual signals on the apron, installation of chocks, and post-flight inspection. Towing is required when the airfield dimensions do not allow the aircraft to taxi under its own power or when it must be moved without crew involvement for maintenance purposes.

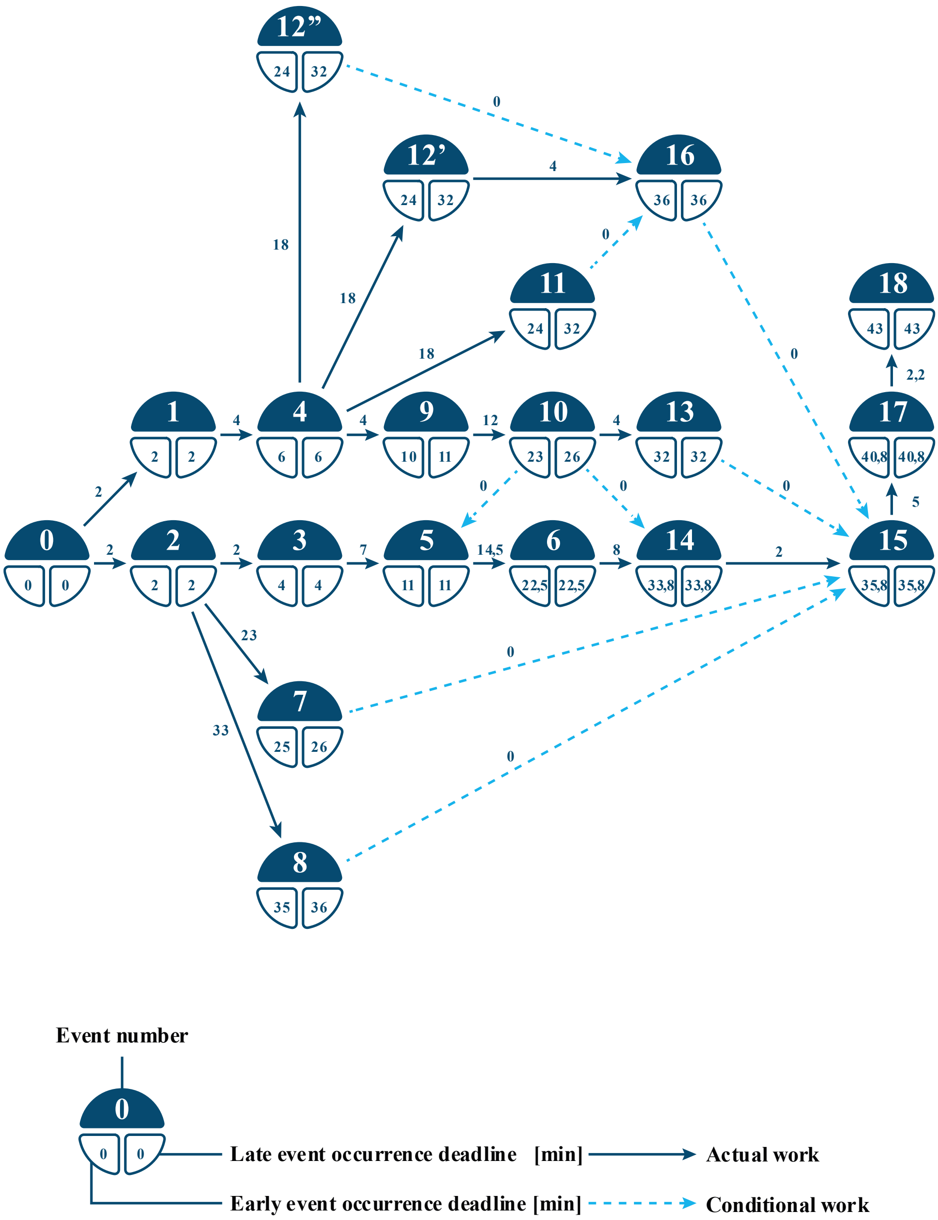
Pre-flight preparation may include: providing ground power supply; cabin air conditioning; loading and refueling; and cleaning. On the stand, the aircraft is often connected to ground power sources to reduce the load on onboard systems. To increase passenger comfort, warm air is pumped into the cabin in winter and cool air in summer. Before departure, fueling with fuel and special fluids is carried out. Cleaning of the cabin and cockpit is done before passenger boarding.

The process flow chart of comprehensive aircraft preparation establishes the standard organization of tasks for executors, aiming to minimize the aircraft's turnaround time to reduce the expected delay duration in individual technological operations.

The flow chart is a time-scaled linear chart plotted in "operation – time" coordinates. The time-scaled linear chart (Table 1.5) contains information on task names and their labor intensity.

The network-based process flow chart defines the chronological sequence of activities involved in the comprehensive pre-flight preparation of the aircraft. It includes data on both the earliest and latest allowable start times for each operation, enabling analysis of time constraints and dependencies.

Connections between events are represented using two types of links: solid lines correspond to actual tasks with defined durations (indicated numerically above the line), while dashed lines represent conditional or logical relationships that do not involve a physical process. These conditional links are used to illustrate dependencies between events where no real-time action occurs, but a sequential constraint still applies.



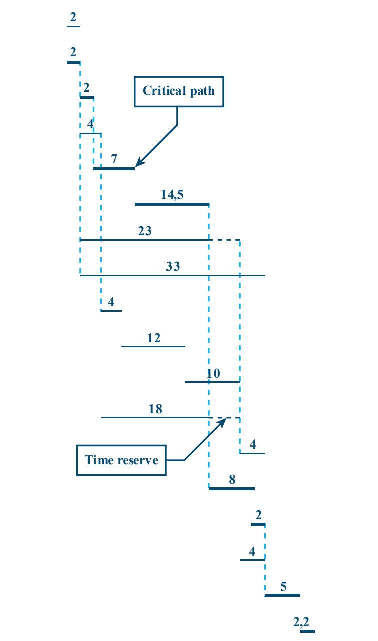
1.3. fig. Flow chart of comprehensive preparation of the aircraft for departure at a transit airport [67].

The parameters of the working technological process shown in Table 1.5 include the names of the tasks involved in this particular maintenance form; the start and end events between which the task is performed; and the duration of the task execution, based on general turnaround procedures outlined in [24].

Table 1.5

Table of working technological process parameters [67]

Event Number	Event Description	Number of performers	Duration of work [min]
1	Establishing crew communication	1	
2	Installing wheel chocks	2	
3	Setting up the boarding stairs	1	
4	Preparatory maintenance work	3	
5	Disembarking passengers	1	
6	Cabin cleaning	12	
7	Unloading and loading baggage	3	
8	Turning on/off onboard power	2	
9	Preparatory fuel refueling work	1	
10	Fuel refueling	1	
11	Water system refueling	1	
12	Route inspection	2	
13	Draining waste	1	
14	Boarding passengers	1	
15	Removing the boarding stairs	1	
16	Final maintenance work	2	
17	Engine start-up	1	
18	Disconnecting power supply	1	
Total time for comprehensive pre-flight preparation T_{sum} = 43 minutes.			



In a network-based process model, the critical path represents the longest sequence of dependent tasks connecting the starting point to the final operation. This path determines the overall duration of the process, as each task on it must be completed on time without any scheduling margin since all have zero float.

For aircraft undergoing pre-flight preparation at a transit airport, the length of the critical path establishes the minimum turnaround time required to complete all essential technical operations prior to departure (see Figure 1.3).

It is important to note that many tasks within the network have variable durations depending on operational conditions. As such, adjusting the execution parameters such as available resources, maintenance efficiency, or staffing can shorten or extend individual task durations. These changes directly affect the total time along the critical path and, consequently, the aircraft's time on the ground.

An event is the result of completing a task. On the network process flow chart, events are represented as circles. For example, in the parameter table, task No. 3 is "installation of the boarding stairs," and the corresponding event is "stairs installed," represented by a circle labeled "3" on the chart. Task No. 3 (see Table 1.5) begins after event No. 2 - "wheel chocks installed" - and ends with event No. 3. The duration of task No. 3 is 2 minutes.

The dispatch for flight includes a pre-flight inspection, monitoring engine start-up, and towing to the start position. Often, the flight crew cannot directly observe the engine condition, so ground personnel monitor the engine start to ensure there are no foreign objects or people near the engines and to oversee the operation of the engines and systems.

When the crew reports no system issues, ground personnel disconnect from the wired communication with the aircraft and switch to visual signaling. The taxi controller gives clearance, the crew requests additional confirmation by visual signal from the release personnel, and if no obstacles are present, the release personnel signals with a gesture granting permission for the aircraft to take off.

In this phase, coordination with air traffic control (ATC) becomes essential to ensure timely clearance and safe integration into the departure flow. As outlined by Nolan, air traffic controllers are responsible for sequencing departures, managing separation, and issuing clearances within defined sectors. Their workload and efficiency directly influence departure punctuality and taxi-out delays [27].

1.8. Determination of the duration of individual technological operations in flight preparation

The time required for refueling is calculated using the formula:

$$T = \frac{q \cdot n \cdot L}{1000 \cdot \gamma \cdot Q}\ [min] \qquad (1.1)$$

where T – duration of refueling operations; L – flight distance; q – fuel consumption per passenger-kilometer; n – number of passengers carried; γ – specific weight of fuel, 0.8; Q – fuel system refueling rate, 7000 liters per minute.

The time required for passenger boarding is calculated using the formula:

$$T_l = \frac{n}{CL}\ [min] \qquad (1.2)$$

where T_l – passenger boarding time; n – number of passengers carried; CL – coefficient accounting for the number of doors used for boarding.

The time required for passenger disembarkation is calculated using the formula:

$$T_o = \frac{n}{CO}\ [min] \qquad (1.3)$$

where T_o – passenger disembarkation time; n – number of passengers carried; CO – coefficient accounting for the number of doors used for disembarkation.

The time required for cabin cleaning is calculated using the formula:

$$T_{cl} = \frac{12 \cdot 14{,}5}{N}\ [min] \qquad (1.4)$$

where T_{cl} – cabin cleaning time; N – number of personnel (cleaners).

The time required for baggage loading is calculated using the formula:

$$T_{load} = \frac{NC \cdot 13}{16}\ [min] \quad (1.5)$$

where T_{load} – baggage loading time; *NC* – number of containers being loaded.

The time required for baggage unloading is calculated using the formula:

$$T_{ofld} = \frac{NC \cdot 13}{16}\ [min] \quad (1.6)$$

where T_{ofld} – baggage unloading time; *NC* – number of containers being unloaded.

Discrepancies between the actual time taken to perform individual technological operations and the established standards can cause flight departure delays from the airport, thereby threatening flight regularity.

1.9. States and transitions of the aircraft

By the term "state" of the aircraft we mean situations that arise when one or more adverse factors affect an aircraft operating under normal flight conditions, leading to a reduction in the flight safety level. In accordance with EASA Part-M and Part-CAMO, continued airworthiness of the aircraft must be ensured through structured maintenance control, proper recordkeeping, and responsibility allocation within approved organizations [28]. The states of the aircraft will be denoted by the index *S*. In the Airworthiness Standards, such situations are considered as cases of aircraft not being in a condition for safe operation and are subject to procedures outlined in the MEL [58]. They are divided into four:

1. Deterioration of flight conditions;
2. Difficult (hazardous) situation;
3. Emergency situation;
4. Catastrophic situation.

Special situations in flight or on the ground are characterized by a combination of aircraft properties and pilots' psychophysiological indicators that differ from the normative, as well as a flight mode differing from the "normal" one. Based on this concept, five possible states of the aircraft can be identified. Let us denote them as S_i, where i = 0, 1, 2, 3, 4. Normal ("standard") flight without risk factors – S_0; Deterioration of flight conditions - S_1; Hazardous situation - S_2; Emergency situation - S_3; Catastrophic situation - S_4.

S_0 – Normal Flight Condition. Flight is conducted under expected operating conditions within the recommended flight envelope.

S_1 – Degraded Flight Condition. Aircraft state characterized by a slight increase in pilot workload or minor deterioration of aircraft stability, controllability, or flight performance, still within safe operational limits.

S_2 – Alert Condition. Aircraft state characterized by a noticeable increase in pilot workload or significant degradation characteristics, stability, and controllability, with one or more parameters exceeding normal operational limits but not reaching emergency or contingency limits. Timely and appropriate crew actions, including immediate adjustment of flight plan or profile, are necessary to prevent escalation.

S_3 – Emergency Condition. Aircraft state marked by a significant increase in pilot workload and severe degradation of flight characteristics, stability, and controllability, reaching or exceeding emergency or contingency limits as defined in operational procedures. Managing this condition requires high pilot skill to prevent catastrophic outcomes.

S_4 – Catastrophic Condition. Aircraft state in which a catastrophic event has occurred or is imminent, where the likelihood of preventing loss of life is minimal or nonexistent.

In the event of an abnormal (non-normal) situation, additional time may be required for troubleshooting, decision-making, and corrective actions, leading to extended aircraft ground time and departure delays [54]. According to Boeing's global statistical summary of commercial jet airplane accidents, system failures and operational errors remain contributing factors in a significant proportion of hull loss and fatal events. The data highlight the continued importance of proactive safety management and technical reliability in reducing the rate of catastrophic outcomes [29].

To minimize such consequences, the FAA emphasizes a system-oriented safety approach that includes proactive hazard identification, functional hazard analysis, and integration of safety objectives early in design and operations. This ensures protection of critical functions even under failure conditions [30].

Complementing this, ICAO recommends the implementation of Safety Management Systems (SMS) as a proactive framework to maintain acceptable safety performance levels through structured risk analysis, hazard monitoring, and continuous improvement in operational environments [31].

Further consideration will be limited to adverse factors related only to aircraft system failures that result in failure states.

Failure states will be classified according to the severity of their consequences as follows:

1. Failure conditions that do not affect flight safety, i.e., they do not adversely impact the operational capabilities of the aircraft or increase crew workload. Failure conditions without complicating flight conditions – S_{NR} (Safe Non-Reported);
2. Failure conditions that slightly reduce the flight safety of the aircraft and require crew actions well within their capabilities, i.e., characterized by a minor reduction in safety margins or functional capabilities, a slight increase in crew workload, or some physical discomfort to passengers – Failure conditions with complicating flight conditions – S_1;
3. Failure conditions that significantly reduce the aircraft's capability or the crew's ability to cope with adverse operating conditions, i.e., there is a noticeable reduction in safety margins or operational capabilities, a significant increase in crew workload, deterioration of conditions for passengers or cabin crew, possibly resulting in injuries – Complex condition – S_2;
4. Failure conditions that reduce the aircraft's capabilities or the crew's ability to cope with adverse conditions to such an extent that:
   - Deterioration of working conditions or excessive workload to the extent that the flight crew cannot be relied upon to perform their tasks accurately and completely;
   - Serious or fatal injuries to a relatively small number of occupants on board who are not part of the flight crew – S_3 – Emergency condition.
5. Failure conditions leading to catastrophic events with multiple fatalities and usually resulting in the loss of the aircraft. In the occurrence of such a situation, preventing the catastrophe is considered practically impossible and requires exceptional skill and dedication from the flight crew – S_4 – Catastrophic conditions.

The presence of a list of permissible aircraft failures for flight allows, in relation to the arising need for aircraft recovery at airports within the airline network, to provide the following classification of acceptable aircraft in-flight conditions:

- S_0 – No detected failures on the aircraft;

- S_1 – The aircraft has at least one malfunction causing a minor degradation of flight conditions, but not endangering flight safety;
- S_2 – The aircraft has at least one malfunction that complicates flight conditions but allows operation of the aircraft up to the next line maintenance;
- S_3 – The aircraft has at least one malfunction potentially endangering flight safety, but allows operation of the aircraft to the base airport;
- S_4 – The aircraft has at least one malfunction that prohibits flight operations.

1.10. Probabilistic indicators used in the qualitative and quantitative analysis of failure states

In the case of quantitative analysis, probabilistic indicators are expressed as acceptable ranges of average probability per flight hour or per flight. These probabilities are used in the design of multi-engine aircraft for the expected operating conditions of the aircraft and crew actions in accordance with the flight operations manual. The following classification is used in regulatory documents:

- Frequent failure conditions (S_{NR}), which do not affect flight safety, i.e., do not adversely impact the operational capabilities of the aircraft and do not increase the crew workload. Their probability of occurrence is $P_{NR} \leq 10^{-3}$ per flight hour (or per flight);
- Probable failure conditions (S_1), which are expected to occur one or more times during the entire service life of an individual aircraft, with a probability of occurrence not exceeding approximately $P_1 < 10^{-4}$ per flight hour (or per flight);
- Remote failure conditions (S_2), are failure states that are unlikely to occur on any single aircraft during its entire operational life but may occur several times over the total operational life of a fleet of aircraft of the same type. The average probability of occurrence is approximately $P_2 < 10^{-5}$ per flight hour (or per flight);
- Extremely remote failure conditions (S_3), are failure states that are not expected to occur on any single aircraft during its entire operational life but may occur a small number of times when considering the total operational life of all aircraft of that type. The average probability of occurrence is $P_3 < 10^{-6}$ per flight hour (or per flight);
- Extremely improbable failure conditions (S_4), are failure states that are not expected to occur during the entire operational life of all aircraft of a given type. The average probability of occurrence is $P_4 < 10^{-7}$ per flight hour (or per flight).

Note that the absence of an approved list of permissible failures leads only to conditions S_0, S_1, S_2, S_{NR}.

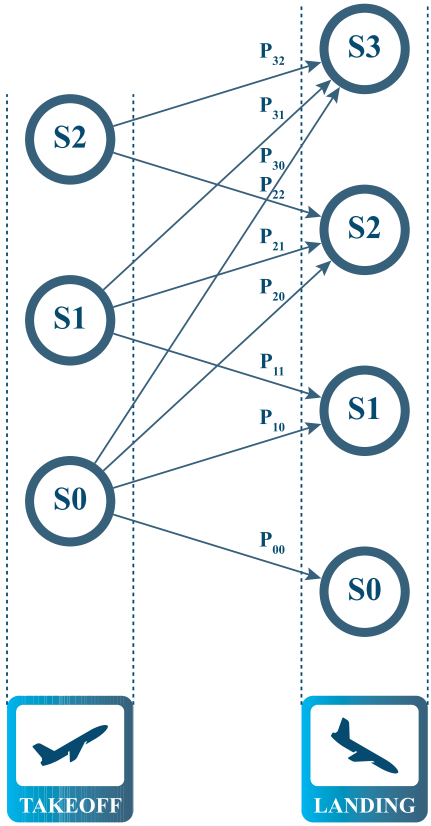
Summarizing the above, we will use the established upper limits of failure condition probabilities per flight as follows:

- For catastrophic failure condition – $P_4 < 10^{-7}$;
- For hazardous failure condition – $P_3 < 10^{-6}$;
- For major failure condition – $P_2 < 10^{-5}$;
- For failure condition with flight conditions complexity – $P_1 < 10^{-4}$;
- For failure condition without flight conditions complexity – $P_{NR} < 10^{-3}$.

Thus, the scheme of possible transitions of the aircraft states S within a single flight can be represented as a graph under the following conditions, see Figure 1.4 (state classification structure is consistent with safety definitions and risk models in [28], [31], and [30]):

- During the flight, the state of the aircraft does not improve;

- Departure on a flight (i.e., from an airport of type B) is permitted only in states S_0, S_1 and S_2;
- Departure on a flight from airports of types T and TB is permitted in states S_0, S_1, S_2 and S_3;



1.4. fig. Flow chart of comprehensive preparation of the aircraft for departure at a transit airport [66].

The state S_{NR} is not included in the graph because it does not affect flight safety, and the aircraft can operate in this state until an appropriate decision is made by the engineering and technical service.

At any stage of the flight within the airline's network, aircraft failures (changes in the aircraft's condition) may occur in flight or on the ground, requiring rectification (aircraft restoration) to enable continuation of the flight. In the context of technical operation, aircraft are categorized as "fit" or "not fit" for continued flight depending on their current technical condition. Restoration actions are required in the event of a "not fit" state [61]. Thus, a change in the aircraft's condition during flight or on the ground necessitates restoration at an airport within the airline's network, which poses a risk of flight delay. The restoration time must be taken into account when evaluating the total downtime of the aircraft [57]. The total restoration duration of the aircraft is described by the restoration time distribution function *F(t)* for the corresponding airport type. If spare parts are not available on site, they are delivered through the available supply channels. The delivery time is modeled by the distribution function *G(t)* for the respective airport type.

To assess the probability of departure delays at airports within an airline's network, it is necessary to develop mathematical models of the aircraft state changes during its time within the operational control loop while performing a flight, based on the approaches described above:

- Model of the aircraft state change during a single flight segment of a route;
- Model of the aircraft state change throughout the entire route;
- Model of the aircraft state change across a sequence of routes over the period of its presence within the operational control loop of the aircraft maintenance process.

1.11. Entropy-Based Theoretical Foundations

An additional theoretical and methodological foundation of this research is the entropy-based approach to assessing the functioning of active systems [68]. Within this approach, the level of uncertainty and informational instability of a system is regarded as a key indicator of its operational effectiveness. In the context of managing the technical operation of aircraft, the transitions between the states S_0, S_1, S_2, and S_3 which describe the degradation of an aircraft's technical condition during flight or on the ground and the need for restoration at airports within the airline's network can be interpreted as changes in the entropy of the system. The higher the probability of transitions to states S_2 and S_3, the higher the level of uncertainty and the greater the operational risks associated with departure delays.

Accordingly, measures aimed at reducing aircraft recovery time (T_R), increasing the probability of spare part availability (P_{SP}), reducing logistical uncertainty (T_D), and decreasing the probability of loop-back events (P_{LB}) represent, in entropy terms, a reduction of informational uncertainty and an increase in controllability of the system.

The practical implementation of this approach is demonstrated in the analysis of the real RAF-AVIA route RIX–TLL–WAW–FRL–SOF–RIX, where decreases in the probabilities of transitions to states S_2 and S_3 are directly interpreted as reductions in the entropy of the aircraft technical-operation process, thereby increasing the predictability of pre-departure procedures and reducing overall expected departure delays. Thus, the entropy-based concept proposed by Goncharenko provides an additional theoretical foundation for interpreting the modelling results and further supports the need to optimize operational aircraft maintenance management to minimize uncertainty and enhance flight regularity.

1.12. IATA Operational Availability Framework

An important component of the theoretical foundation of this research is the set of recommendations issued by the International Air Transport Association (IATA) regarding the assessment of aircraft operational availability. Within the IATA framework, operational availability is defined as an integrated indicator that characterizes the capability of an aircraft to perform scheduled flights under the existing failure rates and established restoration procedures. [69]

According to IATA, aircraft availability is determined by the interaction between two fundamental processes: the occurrence of failures and the restoration of the aircraft to an operable condition. Any increase in restoration time, delays in the delivery of spare parts, shortage of required components, or insufficient maintenance resources leads to a reduction in operational availability and, consequently, an increased likelihood of departure delays. Frequent transitions into non-permissible technical states exacerbate this effect, resulting in a notable decline in the operational readiness of the aircraft.

In the context of the models developed in this dissertation, operational availability can be expressed through the probabilities of the aircraft residing in states S_0, S_1, S_2, and S_3. States S_0 and S_1 correspond to normal or conditionally acceptable readiness levels, whereas transitions to S_2 and especially S_3 indicate a decrease in availability and a corresponding increase in the risk of technical delay. Therefore, the evolution of these state probabilities throughout the route directly reflects the operational availability dynamics described in IATA methodologies.

The parameters considered in this research recovery time (T_R), logistical delivery time (T_D), probability of spare part availability (P_{SP}), and the structure of the route network fully align with the factors that influence operational availability in the IATA classification. As a result, the implementation of the developed models within the operational control system effectively enables the assessment of aircraft availability in accordance with international best practices, and provides the basis for improving flight regularity across the airline network.

2. DEVELOPMENT OF THE AIRCRAFT MAINTENANCE PROCESS MODEL AT THE LEVEL OF OPERATIONAL CONTROL LOOP

2.1. Graphical Model

Structure of the aircraft maintenance process

To describe the process of aircraft state changes on the graph shown in Figure 1.4, given the structure of the airline network, it is necessary to obtain an interconnected description of the aircraft state change processes during a flight, over a route, and over a sequence of routes within the time the aircraft remains in the operational control loop of the maintenance process, taking into account the sequence of aircraft transitions within this operational control loop.

It is evident that the possibilities of such a description are determined by a set of assumptions:

- Regarding the restoration of the aircraft state both within the operational control loop of the maintenance process (i.e., the airline network airport), and outside of this loop;
- Regarding the nature of the processes governing the changes in the state of both individual aircraft in the operating fleet and the entire fleet as a whole.

In accordance with the classification of aircraft states shown in Figure 1.4, schemes of possible transitions of the aircraft states S within the operational control loop of the maintenance process are presented.

The following assumptions regarding the aircraft state transition process follow from Figure 1.4:

- The aircraft state does not improve;
- Departure on a flight (i.e., from a base airport type B) is permitted only in states S_0 and S_1;
- Departure from transit (T) and transit base (TB) airports is permitted in states S_0, S_1 and S_2.

According to Figure 1.4, the distinction between airports based on their aircraft recovery capabilities is as follows:

- At base airports (B), states S_2 and S_3, may be restored to levels S_0 or S_1 depending on the arising operational situation and the characteristics P_{SP}^{B}, $F^{B}(t)$, $G^{B}(t)$, P_3, P_{RES}, T_{AT}^{B};
- At transit (T) and transit base (TB) airports, state S_3 may be restored to S_0, S_1, or S_2 depending on the operational situation and characteristics P_{SP}^{T}, P_{SP}^{TB}, $F^{T}(t)$, $F^{TB}(t)$, $G^{T}(t)$, $G^{TB}(t)$, T_{AT}^{T}, T_{AT}^{TB}. State S_2 at these airports may be restored to S_1 or S_0, or may not be restored at all.

Thus, the differences between transit (T) and transit base (TB) airports essentially reduce to variations in the characteristics P_{SP}, $F(t)$, $G(t)$ and T_{AT}.

Considering that the flight duration is significantly shorter than the aircraft's operational life, it can be assumed that the failure rate parameter remains constant within the timeframe of a single flight.

The model assumes that both the failure rate and repair time stay constant throughout the period being analyzed. System reliability and maintainability can be assessed using probabilistic models and statistical distributions such as Weibull or exponential laws [64]. These principles are essential for making technical decisions in systems where maintenance plays a critical role [32]. According to Boeing's experience, maintenance program enhancements can be achieved by aligning scheduled tasks with real-world reliability data, allowing operators to reduce unnecessary inspections while maintaining safety margins [65].

Typically, the aircraft fleet operated by an airline is distributed across various stages of their service life (such as time between overhauls or approved service life), and the onboard systems and components installed on these aircraft are also at different stages of their respective lifespans.

Therefore, each aircraft, as a potential source of failures, generally differs from others in the fleet.

To account for the specific characteristics of individual aircraft operated by the airline, it is necessary to consider the dependency of the failure rate parameter on time in service, as well as the distribution of aircraft and their components by accumulated flight hours (or cycles).

In principle, such an approach is feasible provided that the airline has an appropriate aircraft maintenance information system (MIS) in place.

However, for the purposes of this study, it is assumed that the aircraft failure rate parameters remain stationary both within the timeframe during which the aircraft operates under the scope of the operational maintenance control system and during the sequence of transitions into and out of this control loop [33].

This assumption is considered reasonable and aligns with the simplified assumptions commonly used in model-based PHM approaches for aircraft systems [34], since accounting for the heterogeneity of the aircraft fleet and their components in terms of time in service, as well as incorporating the dependency of failure rate parameters on accumulated utilization, represents a further and justifiable direction for future development of the proposed model.

According to Airbus's Global Market Forecast, the number of commercial aircraft in service is expected to more than double by 2042, driven by increased passenger demand and fleet modernization. This growth will significantly expand the demand for maintenance services, operational reliability, and advanced fleet management strategies [35].

A direct transition to a higher level of model complexity at this stage could obscure the qualitative understanding of essential properties of the studied process and potentially lead to misinterpretation of the results.

Therefore, building the model under the given assumption (especially considering the absence of such a model and its analysis in the literature available to the author) is regarded as a necessary and mandatory step.

The process of aircraft recovery and the occurrence of flight delays

The recovery of aircraft condition at airports within the airline network and the resulting departure delays caused by the need for such recovery are associated with:

- With the occurrence of the need for aircraft recovery, which is generated by the process of aircraft state transitions described in Section 1.6;
- With the conditions and organization of aircraft recovery at the airline network airports, which can be described using the parameters provided in Table 1.4 (Table structure adapted by the author based on maintenance resource allocation frameworks in [7] and [21]);
- With the status of the Minimum Equipment List (MEL), i.e., the presence of operational states S_1 and S_2.

According to the definition of aircraft operational states provided in Section 1.6, the following regeneration cycles of these states are identified, each associated with the restoration of the aircraft's airworthiness [36]:

- For state S_1, the transition to state S_0 i.e., the restoration of the aircraft's airworthiness occurs during the time intervals between successive entries into the operational maintenance management cycle. This recovery ($S_1 \rightarrow S_0$) is implemented through scheduled maintenance and/or repair activities [37];

- For state S_2, restoration is performed between successive flights, i.e., transitions $S_2 \rightarrow S_0$ or $S_2 \rightarrow S_1$ can only be implemented in an airport of type “B” (Base Airport). If the preceding flight involved a transition $S_0 \rightarrow S_2$, then recovery must follow the $S_2 \rightarrow S_0$ path. However, if the preceding flight involved a transition $S_0 \rightarrow S_1$, then recovery is carried out via $S_2 \rightarrow S_1$, corresponding to the previously described regeneration cycle for state S_1;
- For state S_3, restoration occurs within the time between consecutive departures, i.e., transitions $S_3 \rightarrow S_0$, $S_3 \rightarrow S_1$, or $S_3 \rightarrow S_2$ are carried out during the aircraft’s stay at any of the considered types of airports. The state to which the aircraft transitions depends on the state it was in when departing on its last flight and on the type of airport where the restoration takes place.

The above-described regeneration cycles of aircraft states during their technical operation are presented in Figures 2.1 to 2.3 (recovery logic diagrams conceptually follow the CAMP-based structure in [39] and [24]) as restoration graphs of the aircraft states corresponding to airports of types “B”, “T”, and “TB”, respectively.

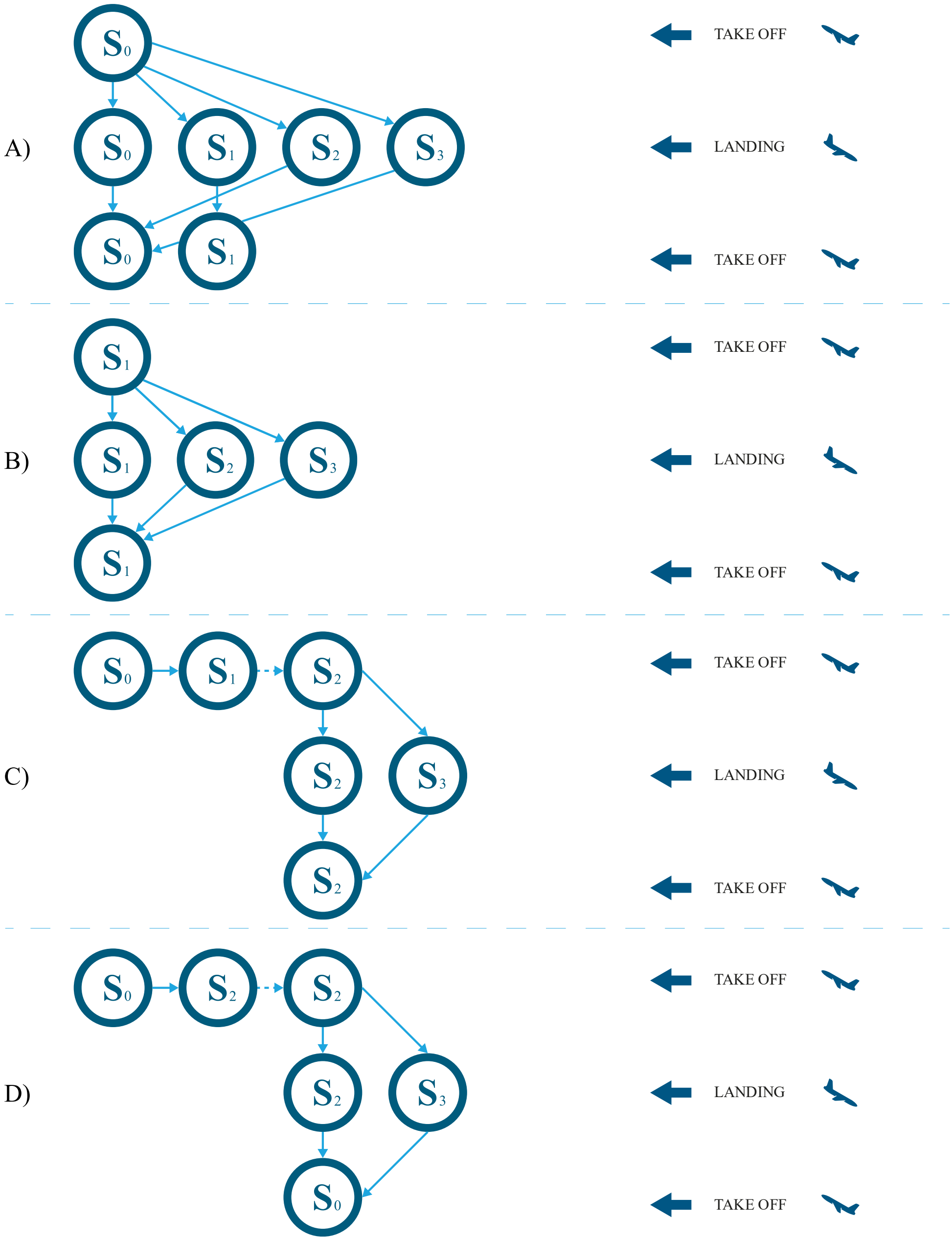
In maintenance planning practice, aircraft management relies on documents such as the Maintenance Planning Document (MPD) and the operator’s Approved Maintenance Program (AMP) [51, 63], which define the intervals, criteria, and scope of scheduled maintenance tasks. For Airbus A320 aircraft, the MPD outlines maintenance checks based on flight hours, flight cycles, or calendar time, ensuring predictable restoration of airworthiness and readiness for operation [38].

Separately, the Continuous Airworthiness Maintenance Program (CAMP) is one of the FAA-approved systems designed to ensure the ongoing reliability of commercial aircraft [39]. The need for aircraft state restoration at the airline network airports is caused by situations leading to flight delays. According to Table 1.4, two main mechanisms of delay formation can be identified.

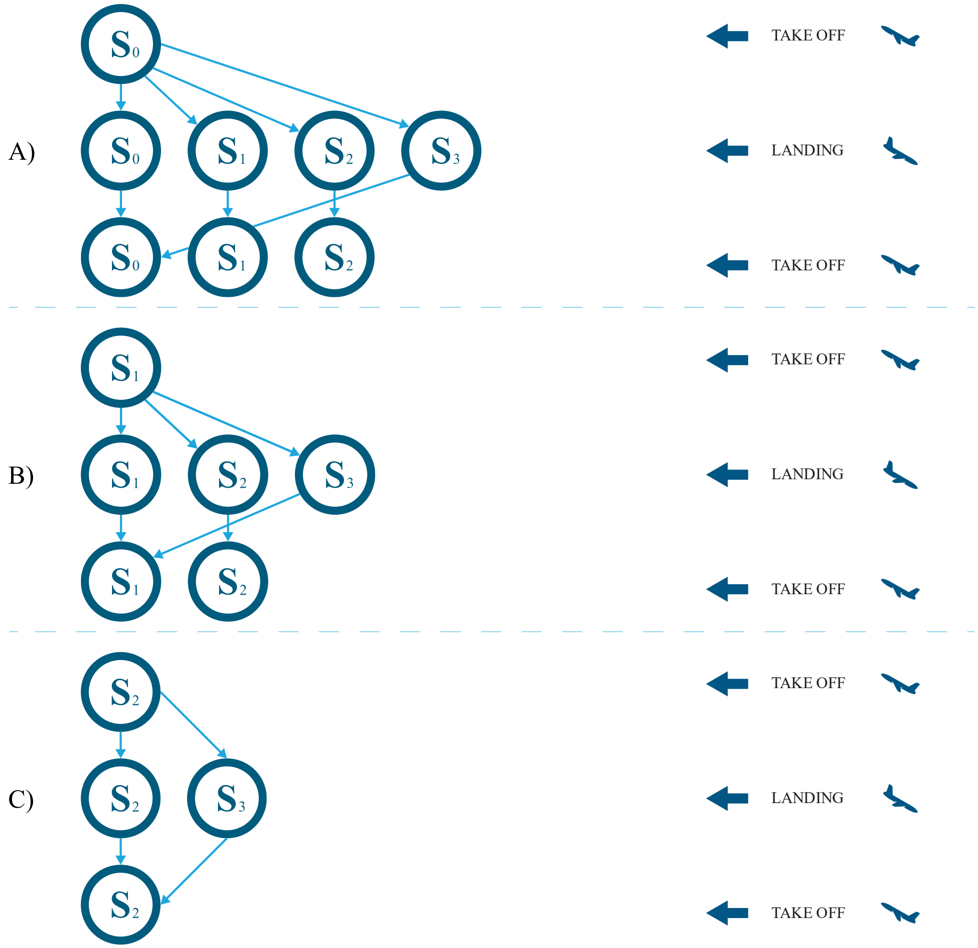
The first of these mechanisms is characteristic of airports of types “T” and “TB” and can be described as follows:

- When the need for aircraft recovery arises, the possibility of its implementation is determined (with probability P_{SP}) by using the necessary spare parts available at the given airport;
- If such spare parts are available, the total recovery duration is described by the recovery time distribution function $F(t)$ corresponding to the airport type;
- If such spare parts are not available, they are delivered through the available channels, with the delivery duration described by the distribution function $G(t)$ corresponding to the airport type. In this case, the total recovery time of the aircraft is the sum of the durations described by the distribution functions $F(t)$ and $G(t)$ for the respective airport type.

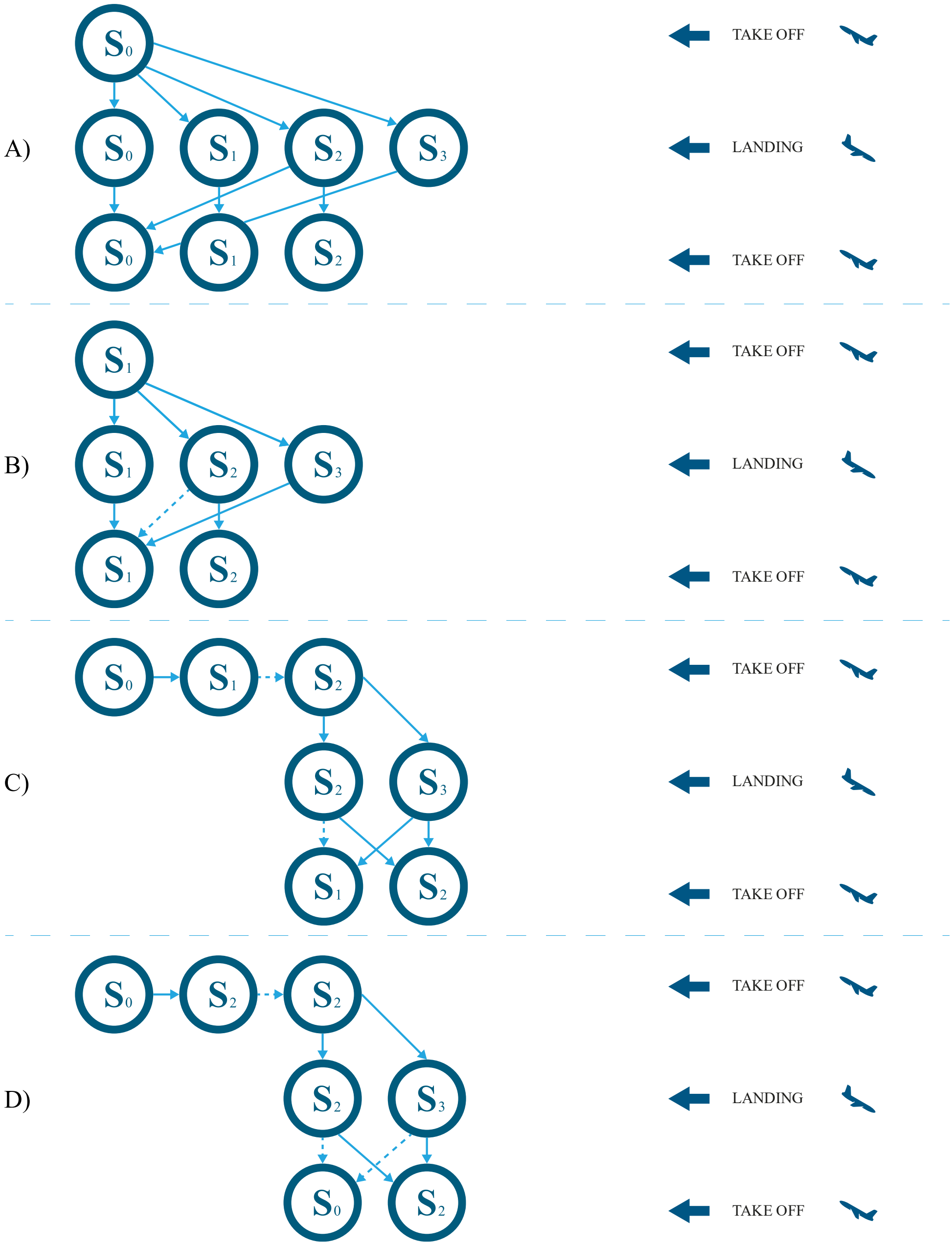
This calculation logic was implemented in Python using basic structured programming techniques for data input, processing, and output formatting [40].



2.1. fig. State and transition graphs of the aircraft in a “B”-type airport and the preceding flight [66].



2.2. fig. State and transition graphs of the aircraft in a "T"-type airport and the preceding flight [66].



2.3. fig. State and transition graphs of the aircraft in a “TB”-type airport and the preceding flight [66].

The second mechanism is characteristic of “B”-type airports and is related to the first by the probability of aircraft loop-back into the next flight.

If loop-back occurs, then unlike the first mechanism, the presence of a reserve aircraft for the flight is determined with probability P_{RES}. If no reserve aircraft is available, the first restoration mechanism is implemented.

If no loop-back of the aircraft into the next flight occurs, it is assumed that situations leading to flight departure delays do not arise.

If the total aircraft recovery time (according to the first and second mechanisms described above) does not exceed the aircraft turnaround time (T_{AT}) at the corresponding type of airport, then no departure delay occurs. Otherwise, the delay duration is determined as the difference between the total recovery time and T_{AT}.

The following assumptions were made in describing the above mechanisms:

- The identification of the failed component (element, assembly, unit) is carried out by the aircraft onboard health monitoring system, which localizes the failure down to the replaceable part;
- Stocks of replaceable parts are maintained at the airline network airports, and prompt delivery of the required parts to the airports where the need arises is possible.

These assumptions reflect current trends in the development of aircraft maintenance support within airlines worldwide.

Using the characteristics of airline network airports presented in Table 1.4, the aircraft recovery graphs (Figs. 2.1–2.3), as well as the above-described delay formation mechanisms, a formalized description of the aircraft recovery processes, and the occurrence of departure delays caused by changes in the aircraft state during flight or on the gound operations can be developed.

2.2. Mathematical model of aircraft condition change per one flight

Let η denote the aircraft state at the start of the flight. The possible values of η (see Figure 1.4) are $\{S_0, S_1, S_2\}$.

Let q_0, q_1, q_2 denote the probabilities that the aircraft is in states S_0, S_1, S_2, respectively, at the beginning of the flight.

This can be expressed as:

$$P\{\eta = S_j\} = q_j; \textstyle\sum_j q_j = 1; j = 0,1,2 \qquad (2.1)$$

The aircraft state at the beginning of the flight is thus a random variable with the set of possible states $\{S_0, S_1, S_2\}$ and a probability distribution (see Figure 1.2).

Let us denote ξ as the state of the aircraft at the end of the flight. The possible values of ξ (see Figure 1.4) are the states $\{S_0, S_1, S_2, S_3\}$.

Let us denote p_0, p_1, p_2, p_3 as the probabilities that the aircraft is in states S_0, S_1, S_2, S_3 respectively at the end of the flight. This is written as:

$$P\{\xi = S_i\} = p_i; \textstyle\sum_i p_i = 1; i = 0,1,2,3 \qquad (2.2)$$

The aircraft’s condition at the end of the flight is a random variable with four possible states S_0, S_1, S_2, S_3 and a corresponding probability distribution (see Figure 1.4).

Let us denote $p_{ij} = P\{\xi = \frac{S_i}{\eta} = S_j\}$ the conditional probability that the aircraft's state at the end of the flight is S_i, given that its state at the beginning of the flight was S_j. These conditional probabilities p_{ij} have the following property:

$$\sum_{i=0}^{3} p_{ij} = 1; j = 0,1,2 \tag{2.3}$$

Some of the values p_{ij}, according to the state transition scheme shown in Figure 1.4, will be equal to zero.

All the listed elements p_{ij} are arranged into a rectangular matrix P of size 4 x 3, which is called the transition matrix or the matrix of transition probabilities describing the change of aircraft states from the beginning to the end of the flight.

$$P = \begin{pmatrix} 1 - p_{10} - p_{20} - p_{30} & 0 & 0 \\ p_{10} & 1 - p_{21} - p_{31} & 0 \\ p_{20} & p_{21} & 1 - p_{32} \\ p_{30} & p_{31} & p_{32} \end{pmatrix} \tag{2.4}$$

The state probabilities q_j and p_j are written in the form of state probability vectors as follows:

$$p = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}, q = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \end{pmatrix} \tag{2.5}$$

Then the relationship between the vectors p and q is given by the formula, which in matrix form is succinctly written as shown in classical Markov reliability modeling frameworks [41]:

$$p = Pq \tag{2.6}$$

Conditional probabilities p_{ij} are considered given and are related to the reliability level of the operated aircraft fleet, as well as to the justification of the list of allowable aircraft failures before reaching a base-type (B) airport and/or the next scheduled maintenance of the aircraft. Thus, the probabilities $(1 - p_{00}) = p_{10} + p_{20} + p_{30}$, $(1 - p_{11}) = p_{21} + p_{31}$, $(1 - p_{22}) = p_{32}$ represent the probabilities of failures occurring during flight or on the ground given that the aircraft departed in states S_0, S_1 and S_2, respectively. The probabilities pp_{00}, p_{11} and p_{22} represent the probabilities of no failures occurring during flight or on the ground given that the aircraft departed in states S_0, S_1, S_2, respectively.

The ratios $\frac{p_{32}}{p_{30}}$, $\frac{p_{31}}{p_{30}}$, $\frac{p_{21}}{p_{20}}$, which by definition are greater than or equal to one, can be considered as indicators of the validity of the list of allowable failures for states S_2 and S_1, respectively. The lists are considered valid if these ratios are equal to 1 (one).

Using known reliability theory metrics, expressed for p_{00}, it can be written in the following form:

$$p_{00} = e^{-\omega \mathrm{T}_{NSF}} \quad (2.7)$$

Where T_{AN} is the average duration of a non-stop flight (*NSF*) of the aircraft in the airline network, and ω is the failure rate parameter of the aircraft, given the aircraft departs in state S_0 .

2.3. Mathematical model of aircraft condition change over a route

Each route consists of a sequence of flights separated by ground time intervals at base airports (type B).

It is assumed that a route consists of N flights. If, at the end of any flight, the aircraft is found to be in state S_3, it must undergo restoration directly at that airport. After such restoration, the route continues. However, a departure delay may occur at the restoration airport.

Let us introduce a random sequence η_n, defined as the state of the aircraft at the beginning of the n-th flight. According to Figure 1.4, the possible values of the random sequence η_n are $\{S_0, S_1, S_2\}$, with $\eta_1 \epsilon \{S_0, S_1\}$.

Furthermore, the random sequence possesses a monotonicity property in the sense that the index i of the current states cannot decrease (see Figure 1.4). This property is expressed as:

$$\text{If } \eta_n = S_j \text{ and } \eta_{n+1} = S_i \text{, then } i \geq j \quad (2.8)$$

The state probabilities of the sequence η_n are determined by the following formulas:

$$P\{\eta_n = S_0\} = q_0(n) \qquad q_0(n) + q_1(n) + q_2(n) = 1$$

$$P\{\eta_n = S_1\} = q_1(n) \qquad q(n) = \begin{pmatrix} q_0(n) \\ q_1(n) \\ q_2(n) \end{pmatrix} \quad (2.9)$$

$$P\{\eta_n = S_2\} = q_2(n)$$

State S_3 is not allowed for the sequence η_n by definition, therefore $P\{\eta_n = S_3\} = q_3(n) = 0$.

The evolution of the process η_n is determined by changes in the state probability vector. Transitions between states for the sequence η_n in one step are defined by the state transitions during a single flight. Specifically, the fact that at the beginning of the $(n + 1)$-th flight the aircraft state is S_0, S_1 or S_2 is equivalent to the fact that at the end of the n-th flight the aircraft state was respectively S_0, S_1 or S_2.

This is because state S_3 is not allowed for the sequence η_n, and during the turnaround at the airport after the n-th flight (i.e., at non-base airports of types "T" and "TB") the aircraft is restored to the state it had at the beginning of the n-th flight in accordance with the transition graphs shown in Figures 2.2–2.3.

This allows us to write the following formulas:

$$q_0(n+1) = \widetilde{p_{00}} \cdot q_0(n) \quad (2.10)$$

$$q_1(n+1) = \widetilde{p_{10}} \cdot q_0(n) + \widetilde{p_{11}} \cdot q_1(n)$$

$$q_2(n+1) = \widetilde{p_{20}} \cdot q_0(n) + \widetilde{p_{21}} \cdot q_1(n) + \widetilde{p_{22}} \cdot q_2(n)$$

These formulas represent the law of total probability, where $\widetilde{p_{ij}}$ are the conditional probabilities of state transitions during a single flight. Taking into account Figures 2.2 and 2.3, we have the following expressions for $\widetilde{p_{ij}}$:

$$\begin{aligned} \widetilde{p_{00}} &= 1 - p_{10} - p_{20} & \widetilde{p_{11}} &= 1 - p_{21} \\ \widetilde{p_{10}} &= p_{10} & \widetilde{p_{21}} &= p_{21} \\ \widetilde{p_{20}} &= p_{20} & \widetilde{p_{22}} &= p_{22} \end{aligned} \tag{2.11}$$

The conditional probabilities $\widetilde{p_{ij}}$ in formulas (2.11) are arranged in a square matrix $\tilde{P}$ of order 3x3:

$$\tilde{P} = \begin{pmatrix} 1 - p_{10} - p_{20} & 0 & 0 \\ p_{10} & 1 - p_{21} & 0 \\ p_{20} & p_{21} & 1 \end{pmatrix} \tag{2.12}$$

The relationship between the vectors $q(n)$ and $q(n+1)$ is expressed by the matrix formula:

$$q = (n+1) = \tilde{P}q(n) \tag{2.13}$$

The introduced random sequence η_n corresponds to a random sequence ξ_n, which is defined as the state of the aircraft at the end of the n-th flight of the considered route.

The possible values of the random sequence ξ_n are the states $\{S_0, S_1, S_2, S_3\}$, with the corresponding probabilities of the aircraft's state defined accordingly. This is expressed as:

$$\begin{aligned} P\{\xi_n = S_0\} &= p_0(n) & p_0(n) + p_1(n) + p_2(n) + p_3(n) &= 1 \\ P\{\xi_n = S_1\} &= p_1(n) & & \\ P\{\xi_n = S_2\} &= p_2(n) & p(n) &= \begin{pmatrix} p_0(n) \\ p_1(n) \\ p_2(n) \\ p_3(n) \end{pmatrix} \\ P\{\xi_n = S_3\} &= p_3(n) & & \end{aligned} \tag{2.14}$$

The sequences η_n and ξ_n are related by an order relation in the sense that the states ξ_n cannot be lower than the states η_n. These relations can be expressed as follows:

$$\text{If } \eta_n = S_j \text{ and } \xi_n = S_i, \text{ then } i \geq j \tag{2.15}$$

Possible state transitions from the sequence η_n to the sequence ξ_n are determined by the possible transitions during a single flight, as shown in Figure 1.4.

The sequences η_n and ξ_n are related not only by the order relations (2.15), but also by a specific connection between the probability distributions of these two processes. Unlike the sequence η_n, the random sequence ξ_n does not possess the property of sequential dependence between the distributions $q(n)$ and $q(n+1)$. According to the law of total probability, we have the following formal relations.

$$\begin{gathered} p_0(n) = p_{00} \cdot q_0(n) \\ p_1(n) = p_{10} \cdot q_0(n) + p_{11} \cdot q_1(n) \\ p_2(n) = p_{20} \cdot q_0(n) + p_{21} \cdot q_1(n) + p_{22} \cdot q_2(n) \\ p_3(n) = p_{30} \cdot q_0(n) + p_{31} \cdot q_1(n) + p_{32} \cdot q_2(n) \end{gathered} \tag{2.16}$$

Where p_{ij} are the conditional probabilities of transition during one flight, introduced and discussed in section 2.2.1. In matrix form, these relations can be written as:

$$P(n) = P \cdot q(n) \tag{2.17}$$

2.4. Mathematical model of aircraft state changes across a sequence of routes

To describe the changes in aircraft states within the operational control loop of the aircraft maintenance process, a random sequence ξ_r is introduced, which is defined as the state of the aircraft at the beginning of the r-th flight sequence (flight leg). The possible values of the random sequence ξ_r are $\{S_0, S_1\}$. Corresponding probabilities of the aircraft states are defined as follows:

$$\begin{aligned} &P\{\xi_r = S_0\} = \pi_0(r) \\ &P\{\xi_r = S_1\} = \pi_1(r) \qquad \pi(r) = \begin{pmatrix} \pi_0(r) \\ \pi_1(r) \end{pmatrix} \\ &\pi_0(r) + \pi_1(r) = 1 \end{aligned} \tag{2.18}$$

States S_2 and S_3 are not allowed at the beginning of flights (flight sequences), and therefore the probabilities of such states are zero, i.e., $P\{\xi_r = S_2\} = 0$, $P\{\xi_r = S_3\} = 0$.

For the process ξ_r, only transitions toward increasing the index i of the state S_i are possible.

At the initial moment $r = 1$, i.e., immediately after the aircraft enters the operational control loop of the maintenance process, the aircraft state is S_0, that is, $\xi_1 = S_0$.

For the analysis of the process ξ_r, the following events are distinguished:

$$A = \{During\ operation\ until\ scheduled\ maintenance, the\ aircraft\ state\ remains\ S_0\}$$
$$\bar{A} = \{During\ operation\ until\ scheduled\ maintenance, aircraft\ transitions\ to\ state\ S_1\}$$

Note that events A and $\bar{A}$ are mutually exclusive and exhaustive, so their probabilities $P\{A\}$ and $P\{\bar{A}\}$ satisfy the relation:

$$P\{A\} + P\{\bar{A}\} = 1$$

To determine the probabilities of these events, it is sufficient to calculate the probability of one of them.

In addition, it is necessary to determine the expected value m_r of the random sequence ξ_r . More precisely, if the numerical values of the random variables ξ_r are interpreted as the indices of the states S_i, then the mathematical expectation $m_r = \mathrm{M}\xi_r$ is well-defined.

The evolution of the process ξ_r is described by the evolution of the probability vector $\pi(r)$ of the states. According to the law of total probability, the following relationships hold:

$$\begin{aligned} \pi_0(r+1) &= \pi_{00} \cdot \pi_0(r) + \pi_{01} \cdot \pi_1(r) \\ \pi_1(r+1) &= \pi_{10} \cdot \pi_0(r) + \pi_{11} \cdot \pi_1(r) \end{aligned} \tag{2.19}$$

Where $\pi_{ij} = P\{\xi_{r+1} = \frac{S_i}{\xi_r} = S_j\}$ – are the conditional transition probabilities for a single flight sequence (i.e., from one flight leg to the next within the operational control loop).

For the conditional probabilities π_{ij} the following relations hold:

$$\sum_i \pi_{ij} = 1, \pi_{ij} \geq 0, j = 0{,}1 \tag{2.20}$$

From Figures 2.1 to 2.3, it follows that the conditional probability $\pi_{01} = 0$. Therefore, based on property (2.20), we have $\pi_{11} = 1$.

The conditional probabilities defined in (2.19) can be arranged into the following transition matrix:

$$\pi = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix} = \begin{pmatrix} 1 - \pi_{10} & 0 \\ \pi_{10} & 1 \end{pmatrix} \tag{2.21}$$

In this matrix, the only unknown parameter is π_{10}, which is subject to determination.

To determine the transition matrix π, we introduce a random sequence $\widetilde{\eta_n}$, defined as the sequence indicating, at the beginning of the n-th flight, either the absence of failures or the detection of failures of type S_1.

Therefore, the possible values of the sequence $\widetilde{\eta_n}$ are the states $\{S_0, S_1\}$.
According to Figures 2.1–2.3 and the definition of state S_1 (see Section 2.1.3), the sequence $\widetilde{\eta_n}$ has the property of monotonicity:

$$\text{If } \widetilde{\eta_n} = S_j \text{ and } \widetilde{\eta_{n+1}} = S_i \text{, then } i \geq j \tag{2.22}$$

The state probabilities of the sequence $\widetilde{\eta_n}$ are determined by the formulas:

$$P\{\widetilde{\eta_n} = S_0\} = \widetilde{q_0}(n) \tag{2.23}$$

$P\{\widetilde{\eta_n} = S_1\} = \widetilde{q_1}(n)$

$\widetilde{q_0}(n) + \widetilde{q_1}(n) = 1$

$$\tilde{q}(n) = \begin{pmatrix} \widetilde{q_0}(n) \\ \widetilde{q_1}(n) \end{pmatrix}$$

The transitions for the sequence $\widetilde{\eta_n}$ are determined by the state transitions over a single flight. According to the law of total probability, the following formulas hold:

$$\widetilde{q_0}(n+1) = \widetilde{q_{00}} \cdot \widetilde{q_0}(n)$$
$$\widetilde{q_1}(n+1) = \widetilde{q_{10}} \cdot \widetilde{q_0}(n) + \widetilde{q_{11}} \cdot \widetilde{q_1}(n) \qquad (2.24)$$

Where the values $\widetilde{q_{ij}}$ are the conditional probabilities of transition over one flight, for which the following expressions hold (see Figure 1.4):

$$\widetilde{q_{00}} = 1 - p_{10};\ \widetilde{q_{10}} = p_{10};\ \widetilde{q_{11}} = 1 \qquad (2.25)$$

The conditional probabilities $\widetilde{q_{ij}}$ in formulas (2.24) are arranged in a square matrix $\tilde{Q}$ of order 2x2:

$$\tilde{Q} = \begin{pmatrix} 1 - p_{10} & 0 \\ p_{10} & 1 \end{pmatrix} \qquad (2.26)$$

The relationship between the vectors $\tilde{q}(n)$ and $\tilde{q}(n+1)$ is expressed by the matrix formula:

$$\tilde{q}(n+1) = \tilde{Q}\tilde{q}(n) \qquad (2.27)$$

Now we can find the transition matrix π for the process ξ_r. Let $X = (X_1, X_2, \dots, X_L)$ be the set of flight types with probabilities $P = (P_1, P_2, \dots, P_L)$ that the aircraft will be assigned to the corresponding flight type.

Let $(N_1, N_2, \dots, N_L)$ be a sequence of L integers defining the number of flights in the corresponding route. Then the matrix π is determined by the formula.

$$\pi = \sum_{l=1}^{L} p_l \cdot \tilde{Q}^{N_l} \qquad (2.28)$$

Where $\tilde{Q}^{N_l}$ denotes the N_l-th power of the square matrix $\tilde{Q}$.

Note that the introduction of the process $\widetilde{\eta_n}$ and its probabilistic characteristics in the form of the matrix $\tilde{Q}$ is related to the necessity of finding the matrix π using formula (2.28). The change in the state probabilities of the process ξ_r is expressed in matrix form as:

$$\pi(r+1) = \pi \cdot \pi(r) \qquad (2.29)$$

From formula (2.29), it follows that the state probability vector $\pi(r+1)$ at the beginning of the $(r+1)$-th flight is determined by the formula:

$$\pi(r+1) = \pi^r \cdot \pi(1) \tag{2.30}$$

Where π^r is the r-th power of the matrix π, and $\pi(1)$ is the state probability vector at the beginning of the first flight. Based on the conditions of full restoration of the aircraft at the start of the first flight, it is assumed that:

$$\pi(1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.31}$$

If in formula (2.29) we set $r = R - 1$, then using the thus obtained state probability vector $\pi(R)$, the probability of event A, considered above, can be found by the formula:

$$P\{A\} = \pi_0(R), P\{\bar{A}\} = 1 - \pi_0(R) = \pi_1(R), \pi(R) = \begin{pmatrix} \pi_0(R) \\ \pi_1(R) \end{pmatrix} \tag{2.32}$$

The mathematical expectation m_r is found as follows:

$$m_r = \pi_1(r) \tag{2.33}$$

In addition to the random process ξ_r, which defines the aircraft state at the start of each route, a random process θ_r is introduced, representing the flight type with index l. The set of possible values of the sequence θ_r is the set $X = \{X_1, X_2, \dots, X_L\}$, corresponding to the flight types in the airline network, or more simply, the index l from the set $\{1, 2, \dots, L\}$. The random variables are defined by the vector $P(r) = (P_l(r))$, where $l = 1, 2, \dots, L$, representing the probabilities (frequencies) of flights within the airline network.

$$\begin{gathered} P_l(r) = P\{\theta_r = X_l\}, P_l(r) \geq 0 \\ \sum_{l=1}^{L} p_l(r) = 1, r = 1, 2, \dots, R \end{gathered} \tag{2.34}$$

The sequence $\{\theta_r,\ r = 1, 2, \dots, R\}$ is considered as a sequence of independent and identically distributed random variables with distribution $P = (P_l)$, which depends on r.

$$P_l = P\{\theta_r = X_l\}, l = 1, 2, \dots, L \tag{2.35}$$

In practice, however, such independence may not hold. The generalization of distribution (2.35) to a dependent random variable is taken as the sequential determination of probability distribution vectors $P(r)$ using the matrix formula:

$$P(r+1) = P^{*}P(t) \tag{2.36}$$

Where $P^{*} = (P_{ij}^{*})$ is the transition probability matrix, defined as the matrix of conditional probabilities that the random variable θ_{r+1} takes the value X_i given that the random variable θ_r took the value X_j. This is expressed by the formula:

$$P_{ij}^{*} = P\{\theta_{r+1} = \frac{X_i}{\theta_r} = X_j\} \tag{2.37}$$

The transition probability matrix $P^{*} = (P_{ij}^{*})$ is assumed to be known in advance. The elements P_{ij}^{*} are non-negative numbers satisfying the conditions:

$$\sum_{i=1}^{L} P_{ij}^{*} = 1\,, j = 1, 2, \dots, L \tag{2.38}$$

If the sequence θ_r represents a sequence of independent random variables, then the transition probability matrix P^{*} can be represented as the product:

$$P^{*} = \begin{pmatrix} P_1 & 0 & 0 & \dots & 0 \\ 0 & P_2 & 0 & \dots & 0 \\ 0 & 0 & P_3 & \cdots & 0 \\ \dots & \dots & \dots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & P_L \end{pmatrix} \cdot \begin{pmatrix} P_1 & 0 & 0 & \dots & 0 \\ 0 & P_2 & 0 & \dots & 0 \\ 0 & 0 & P_3 & \cdots & 0 \\ \dots & \dots & \dots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & P_L \end{pmatrix} = (Diag(P_l)) \cdot I \tag{2.39}$$

Where $(Diag(P_l))$ is a diagonal matrix with the elements P_l from (2.35) on the diagonal, and I is a matrix consisting entirely of ones. The non-negativity property and condition (2.38) for the elements of the matrix P^{*} in (2.39) are obviously satisfied.

The introduced random sequence ξ_r corresponds to the random sequence $æ_r$, which is defined as the state of the aircraft at the end of the r-th flight.

The possible values of the random sequence $æ_r$ are the states $\{S_0, S_1, S_2, S_3\}$, and the corresponding probabilities of the aircraft state are defined. This is expressed as:

$$P\{æ_r = S_0\} = \rho_0(r) \qquad P\{æ_r = S_2\} = \rho_2(r)$$

$$P\{æ_r = S_1\} = \rho_1(r) \qquad P\{æ_r = S_3\} = \rho_3(r)$$

$$\rho(r) = \begin{pmatrix} \rho_0(r) \\ \rho_1(r) \\ \rho_2(r) \\ \rho_3(r) \end{pmatrix} \tag{2.40}$$

$$\rho_i(r) \geq 0 \qquad \rho_0(r) + \rho_1(r) + \rho_2(r) + \rho_3(r) = 1$$

The sequences $æ_r$ and ξ_r are related by order relations in the sense that the states of $æ_r$ cannot be lower than the states of ξ_r. These relations can be written as follows:

$$\text{If } \xi_r = S_j \text{ and } æ_r = S_i\text{, then } i \geq j \tag{2.41}$$

The sequences $æ_r$ and ξ_r are related not only by the order relations (2.41), but also by a specific relationship between the probability distributions of these two processes. Let the vectors $\pi(r)$ and $\rho(r)$ denote the probability distributions of the sequences ξ_r and $æ_r$, respectively. According to the law of total probability, we have:

$$
\begin{aligned}
\rho_0(r) &= r_{00} \cdot \pi_0(r) \\
\rho_1(r) &= r_{10} \cdot \pi_0(r) + r_{11} \cdot \pi_1(r) \\
\rho_2(r) &= r_{20} \cdot \pi_0(r) + r_{21} \cdot \pi_1(r) \\
\rho_3(r) &= r_{30} \cdot \pi_0(r) + r_{31} \cdot \pi_1(r)
\end{aligned}
\tag{2.42}
$$

Where $r_{ij} = P\{æ_r = \frac{S_i}{\xi_r} = S_j\}$, for $i = 0, 1, 2, 3$ and $j = 0,1$, are the conditional probabilities that the aircraft state at the end of the r-th flight will be S_i, given that its state at the beginning of the r-th flight was S_j.

The values r_{ij} satisfy the following conditions:

$$r_{ij} \geq 0, \sum\nolimits_i r_{ij} = 1$$

Note that in the first formula (2.42) there is only one term, unlike the other formulas. This is due to the structure of possible transitions from the aircraft's state at the beginning of the flight to its state at the end of the flight.

The conditional probabilities r_{ij} are arranged into a rectangular transition matrix R^* of order 4×2, which has the following form:

$$R^* = \begin{pmatrix} r_{00} & 0 \\ r_{10} & r_{11} \\ r_{20} & r_{21} \\ r_{30} & r_{31} \end{pmatrix}, \pi(r) = \begin{pmatrix} \pi_0(r) \\ \pi_1(r) \end{pmatrix}, \rho(r) = \begin{pmatrix} \rho_0(r) \\ \rho_1(r) \\ \rho_2(r) \\ \rho_3(r) \end{pmatrix} \tag{2.43}$$

The relationship between the probability vectors $\pi(r)$ and $\rho(r)$, which represent the state distributions of the sequences ξ_r and $æ_r$, respectively, is expressed as follows:

$$\begin{pmatrix} \rho_0(r) \\ \rho_1(r) \\ \rho_2(r) \\ \rho_3(r) \end{pmatrix} = \begin{pmatrix} r_{00} & 0 \\ r_{10} & r_{11} \\ r_{20} & r_{21} \\ r_{30} & r_{31} \end{pmatrix} \cdot \begin{pmatrix} \pi_0(r) \\ \pi_1(r) \end{pmatrix}$$

In matrix form, these relationships are concisely written as:

$$\rho(r) = R^{*}\pi(r) \tag{2.44}$$

For the pair of processes ξ_r and $æ_r$, the relationship (2.44) holds; however, it is not possible to establish a direct relationship between the state probabilities $\rho(r+1)$ and $\rho(r)$ for the sequence $æ_r$ independently of the probability distributions of the sequence ξ_r.

Note that the matrix R^{*} is unknown, and its elements need to be determined. This can be done based on the description of the sequence of flights within a single route.

Let us analyze the first flight of the route. The initial states of the aircraft at the start of the route, or at the beginning of the first flight of this route, are the states $\{S_0, S_1\}$. In this case, the random variables satisfy the equality $\xi_r = \eta_1$, and consequently, the following equalities hold for the probability distributions:

$$P\{\xi_r = S_0\} = P\{\eta_1 = S_0\} = \pi_0(r) = q_0(1)$$

$$P\{\xi_r = S_1\} = P\{\eta_1 = S_1\} = \pi_1(r) = q_1(1) \tag{2.45}$$

$$P\{\eta_1 = S_2\} = q_2(1) = 0$$

For the first flight in the route, along with formulas (2.10), the following relations can be written:

$$q_0(2) = \widetilde{p_{00}} \cdot \pi_0(r)$$

$$q_1(2) = \widetilde{p_{10}} \cdot \pi_0(r) + \widetilde{p_{11}} \cdot \pi_1(r) \tag{2.46}$$

$$q_2(2) = \widetilde{p_{20}} \cdot \pi_0(r) + \widetilde{p_{21}} \cdot \pi_1(r)$$

Where $\widetilde{p_{\iota\jmath}}$ are the conditional probabilities introduced in (2.11). From the conditional probabilities in formulas (2.46), a transition rectangular matrix $\tilde{P}'$ of order 3x2 is formed:

$$\tilde{P}' = \begin{pmatrix} 1 - p_{10} - p_{20} & 0 \\ p_{10} & 1 - p_{21} \\ p_{20} & p_{21} \end{pmatrix}$$

At the same time, formulas (2.46) can be written in matrix form as:

$$q(2) = \tilde{P}'\pi(r) \tag{2.47}$$

Now it is possible to find the transition matrix R^{*} that links the probability distributions $\rho(r)$ and $\pi(r)$ of the aircraft states at the end and the beginning of the r-th flight, respectively. By analogy with the derivation of formula (2.23) and with the corresponding notations for the matrix, the following formula holds:

$$R^* = P \sum_{l=1}^{L} p_l \tilde{Q}^{N_{l-2}} \tilde{P}' \quad (2.48)$$

Thus, the matrices R^* and π, which are used to analyze the processes of aircraft state changes across the sequence of flights, are fully defined.

2.5. Models of aircraft recovery processes and departure delays

The aircraft recovery process consists of restoring failed components, with or without replacement. We will consider the actual recovery time of the aircraft, denoted by τ, as a random variable with its own probability distribution.

If the spare part required for aircraft recovery is available at the airport, then the time τ is represented as a random variable with the distribution function $F(t) = P\{\tau \leq t\}, t \geq 0$.
Depending on the type of the airport under consideration, the distribution functions for the recovery time τ will differ. Therefore, there are three distribution functions: $F^T(t), F^{TB}(t)$ and $F^B(t)$.

In practice, it can be difficult to determine the characteristics of time in the form of distribution functions. Therefore, numerical characteristics, such as the mathematical expectation (mean) $\bar{\delta}$, are often used. It is clear that for different types of airports, these numerical characteristics will vary and require experimental evaluation.

If the required spare part is not available at the airport of landing, it can be delivered promptly to the aircraft. Let the delivery time of the component be denoted by ν. The variable ν is a random variable with the distribution function $G(t) = P\{\nu \leq t\}, t \geq 0$.

Depending on the type of airport, the random variable ν has its own distribution function $G(t)$. Therefore, there are effectively three distribution functions: $G^T(t), G^{TB}(t)$ and $G^B(t)$.

In practice, it is often difficult to determine the delivery time characteristics in the form of a distribution function. Usually, numerical characteristics such as the expected value $\bar{\nu}$ and variance $\bar{\sigma}$ are used instead. These numerical characteristics are statistically estimated for each airport type "T", "TB", and "B."

For the purposes of the current task, the total time spent on aircraft restoration is of interest. Depending on whether the required spare part is available at the given airport or not, this time T will be determined by one of the following equations:

$$T = \tau, T = \tau + \nu \quad (2.49)$$

The calculation of each of the equalities (2.49) is performed with probabilities P_{SP}^T, P_{SP}^{TB} and P_{SP}^B depending on the type of airport, or with their complementary probabilities $1 - P_{SP}^T$, $1 - P_{SP}^{TB}$ and $1 - P_{SP}^B$ respectively.

We will limit ourselves to the numerical characteristics of the random variable T, related only to the first two moments the mean value and the variance. Let χ be a random variable taking two values {0, 1}, characterizing the presence or absence of the replaceable component at the airport, respectively.

If P_{SP} is the probability that the necessary spare part is available at the airport, then the probability value $1 - P_{SP}$ corresponds to its absence. Thus, the pair of numbers $\{1 - P_{SP}, P_{SP}\}$ forms the probability distribution for the discrete random variable χ.

In fact, three random variables χ^T, χ^{TB} and χ^B are considered, depending on the airport type. Each of these three random variables is defined by its corresponding probability distribution.

$$\{1 - P_{SP}^{T}, P_{SP}^{T}\}, \{1 - P_{SP}^{TB}, P_{SP}^{TB}\}, \{1 - P_{SP}^{B}, P_{SP}^{B}\}$$

Using these notations for the analysis of the random variable T, the conditional mathematical expectation $m(\chi)$ of the random variable T, given that the value of the random variable χ is respectively 1 or 0, is defined by the formula:

$$m(\chi) = M\left(\frac{T}{\chi}\right) = \begin{cases} \bar{\tau} & , \chi = 1 \\ \bar{\tau} - \bar{\nu} & , \chi = 0 \end{cases} \tag{2.50}$$

Instead of a single conditional mathematical expectation (2.50), there are actually three conditional mathematical expectations considered, depending on the type of airport where the restoration takes place.

Similarly, conditional variances of the considered random variables are introduced. Assuming independence of the random variables τ and ν, the conditional variance of the random variable T, given a certain value 1 or 0 of the random variable χ, is defined by the formula:

$$D\left(\frac{T}{\chi}\right) = d(x) = \begin{cases} D\tau + \bar{\delta} \\ D(\tau + \nu) = \bar{\delta} + \bar{\sigma} \end{cases} \tag{2.51}$$

Just like with the conditional mathematical expectation, instead of a single conditional variance (2.51), three conditional variances are considered for each type of airport.

The introduced random variables τ and ν allow us to define the time φ the delay time of the aircraft at the airport. This time is a random variable defined by the formula:

$$\varphi = \begin{cases} 0 & , T \le T_{AT} \\ T - T_{AT} & , T > T_{AT} \end{cases} \tag{2.52}$$

Where T_{AT} is the average parking time of the aircraft at the airport (see Table 1.4).
Taking into account expressions (2.49) and (2.51), formula (2.52) can be written as follows:

$$\varphi = \begin{cases} 0 & , T \le T_{AT} \\ \tau + (1 - \chi)\nu & , T > T_{AT} \end{cases} \tag{2.53}$$

Accordingly, the expected value $\bar{\varphi}$ and the variance d of the delay time are determined by the following formulas:

$$\bar{\varphi} = M\varphi = P_{SP}\bar{\tau} + (1 - P_{SP})(\bar{\tau} + \bar{\nu}) = \bar{\tau} + (1 - P_{SP})\bar{\nu} \tag{2.54}$$

$$d = D\varphi = P_{SP}\bar{\delta} + (1 - P_{SP})\left(\bar{\delta} + \bar{\sigma}\right) = \bar{\delta} + (1 - P_{SP})\bar{\sigma} \tag{2.55}$$

Instead of a single average delay time $\bar{\varphi}$ and delay variance d, there are three corresponding values of these quantities for each type of airport in the airline network.

The models of recovery, delivery, and delay processes discussed above need to be linked with the sequence of flights during a route, i.e., they should be considered in connection with the flight execution process and the resulting process of changes in the aircraft states within the set $S = \{S_0, S_1, S_2, S_3\}$.

Consider a sequence of flights with landings at intermediate airports, which are of two types "T" and "TB." Let φ_1, φ_2, … , φ_{N-1} be a sequence of random delay times that can occur at the landing airports. This means that if recovery of the aircraft is required at the landing airport A_i of the corresponding type, then it will take time φ_i. The time φ_i is a random variable with its own time-related characteristics.

Let l_1, l_2, … , l_{N-1} be a sequence of random variables indicating the necessity of aircraft recovery at the corresponding landing airport A_i. The random variables l_i take values from the set {0, 1} with probabilities $(1 - p(i))$ and $p(i)$, respectively.

This is expressed as:

$$P\{l_i = 1\} = P(i), P\{l_i = 0\} = 1 - P(i) \tag{2.56}$$

The sequence of delay times T_{DS1}, T_{DS2}, … , T_{DSN-1} is defined by the formula:

$$T_{DSn} = l_n \cdot \varphi_n, n = 1, 2, \dots, N - 1 \tag{2.57}$$

The meaning of the sequence $\{T_{DSn}\}$ is that it represents the actual delay time of the flight, taking into account the presence of such a delay. If there is no delay, then the value of T_{DSn} is zero. If a delay occurs, then T_{DSn} equals the actual delay time φ_n, which takes place according to formula (2.53).

In terms introduced in section 1.6, the relationship between the sequence of random variables ξ_n, representing the aircraft state at the arrival airport A_n, and the probability distribution of the sequence of indicesl_n is expressed by the formula:

$$P\{l_n = 1\} = P\{\xi_n = S_3\} = P_3(n) \tag{2.58}$$

This is related to the classification of aircraft states: only when the aircraft is in state S_3 does it require restoration at the given arrival airport during the flight time.

Note that the mathematical expectation or mean value of the random variable T_{DSn} can be found using the formula:

$$MT_{DSn} = \overline{\varphi_n} \cdot p_{DSn} = (\overline{\tau_n} + (1 - P_{SP})\overline{\vartheta_n}) \cdot p_{DSn} \tag{2.59}$$

And the variance of the delay time DT_{DSn} is given by the formula:

$$DT_{DSn} = \overline{d_n} \cdot p_{DSn} = (\overline{\delta_n} + (1 - P_{SP})\overline{\sigma_n}) \cdot p_{DSn} \tag{2.60}$$

Here, $\overline{\varphi_n}$, $\overline{\tau_n}$, $\overline{\vartheta_n}$, $\overline{d_n}$, $\overline{\delta_n}$, $\overline{\sigma_n}$ denote the numerical characteristics of the delay times, recovery times, and spare parts delivery times at the arrival airport A_n.

Alongside the sequence $\{T_{DSn}\}$, a sequence of cumulative delay times $\{V_n\}$ over n flights is introduced by the formula:

$$V_n = T_{DS1} + T_{DS2} + \dots + T_{DSn} = \sum_{k=1}^{n} T_{DSk}\,, n = 1,2, \dots, N-1$$

The random variable V_n represents the total delay time during the flight in intermediate airports of type “T” and “TB”, taking into account the actual need for aircraft recovery.

The expected value of the cumulative delay time is calculated as:

$$MV_n = \sum_{k=1}^{n} P_{LB}(k) \cdot \overline{\varphi_k} \tag{2.61}$$

And its variance is calculated as:

$$DV_n = \sum_{k=1}^{n} P_{LB}(k) \cdot \overline{d_k} \tag{2.62}$$

In particular, when $n = N - 1$, these formulas provide the mean value and the variance of the total delay time in all intermediate airports of the given flight.

The considered airline network includes L types of routes. Each route consists of N_l flights, where $l = 1,2, \dots, L$.

Each route type has its own utilization probability p_l. For each route type, there are specific state probability values $p^l(n) = (p_i^{(l)}(n))$, as well as numerical characteristics $\overline{\varphi_n}^{(l)}$ and $\overline{d_n}^{(l)}$ of delay time for the l-th route type.

Let M and D denote the mean and variance of the delay time on the airline network at airports of types "T" and "TB." These numerical characteristics are calculated by the formulas:

$$M = \sum_{l=1}^{L} P_l \sum_{n=1}^{N_l - 1} p_{LB}^{(l)}(n) \cdot \overline{\varphi_n}^{(l)} \tag{2.63}$$

$$D = \sum_{l=1}^{L} P_l \sum_{n=1}^{N_l - 1} p_{LB}^{(l)}(n) \cdot \overline{d_n}^{(l)} \tag{2.64}$$

To define such random variables $\{\Psi_r^{(LB)}\}$, the probability P_{LB} of looping (see Table 1.4) of the aircraft is set. This is expressed as:

$$P\left\{\Psi_r^{(LB)} = 1\right\} = P_{LB}, P\left\{\Psi_r^{(LB)} = 0\right\} = 1 - P_{LB} \tag{2.65}$$

Also introduced is the sequence $\{\Psi_r^{(R)}\}$, which indexes the absence or presence of a reserve aircraft at the base airport. More precisely, $\{\Psi_r^{(R)}\}$ is a sequence of random variables taking two values $\{0, 1\}$, numerically representing the presence or absence of a reserve aircraft for continuing the next flight. To define such random variables $\{\Psi_r^{(R)}\}$, the probability P_{RES} of having a reserve aircraft is set. This is expressed as:

$$P\left\{\Psi_r^{(R)} = 1\right\} = P_{RES}, P\left\{\Psi_r^{(R)} = 0\right\} = 1 - P_{RES} \tag{2.66}$$

In addition, it is necessary to specify the numerical characteristics of the time required to allocate (wait for) the reserve aircraft for conducting the next flight. Let $\bar{\alpha}$ denote the average waiting time for the reserve aircraft, and $\bar{\beta}$ denote the variance of this time. The mean time and variance of the aircraft delay due to waiting for the reserve aircraft are calculated by the following formulas:

$$MW_r = (1 - P_{RES}) \cdot \bar{\alpha}, DW_r = (1 - P_{RES}) \cdot \bar{\beta} \tag{2.67}$$

Here, W_r is the random component of the flight delay time at the base airport due to waiting for the reserve aircraft.

Formulas (2.65) and (2.67) are used to calculate the values of φ for the base airport.

In addition to calculating the mentioned mean values and variances of delay times, to describe the dynamics of their formation during the aircraft maintenance process, it is necessary to know the probabilities of certain events related to the emergence of situations that create conditions for these delays.

For airports of types "T" and "TB" (intermediate airports), such events correspond to the aircraft states from the set $S = \{S_0, S_1, S_2, S_3\}$ occurring upon aircraft landing at these airports during the flight execution.

$$A_0 = A_0(r) = \left\{\begin{array}{c}\text{During the entire } r\text{-th} \\ \text{flight, the aircraft} \\ \text{state will be } S_0.\end{array}\right\} = \left\{\begin{array}{c}\text{During the stop at} \\ \text{any intermediate} \\ \text{airport of the } r\text{-th} \\ \text{flight, the aircraft} \\ \text{state will be } S_0.\end{array}\right\}$$

$$A_1 = A_1(r) = \left\{\begin{array}{c}\text{Throughout the entire} \\ r\text{-th flight, the aircraft} \\ \text{state will be either } S_0 \\ \text{or } S_1.\end{array}\right\} = \left\{\begin{array}{c}\text{During the} \\ \text{turnaround at any} \\ \text{intermediate airport} \\ \text{of the } r\text{-th flight, the} \\ \text{aircraft state will be} \\ \text{either } S_0 \text{ or } S_1.\end{array}\right\}$$

$$A_2 = A_2(r) = \left\{\begin{array}{c}\text{During the entire } r\text{-th} \\ \text{flight, the aircraft} \\ \text{state will be } S_0, S_1, \text{ or} \\ S_2.\end{array}\right\} = \left\{\begin{array}{c}\text{During the} \\ \text{turnaround at any} \\ \text{intermediate airport} \\ \text{of the } r\text{-th flight, the} \\ \text{aircraft state will be} \\ S_0, S_1, \text{ or } S_2.\end{array}\right\} = \left\{\begin{array}{c}\text{Throughout the} \\ \text{entire } r\text{-th flight,} \\ \text{the aircraft state} \\ \text{will not require any} \\ \text{maintenance at} \\ \text{intermediate} \\ \text{airports.}\end{array}\right\}$$

$$\bar{A}_2 = \bar{A}_2(r) = \left\{\begin{array}{c}\text{Throughout the entire} \\ r\text{-th flight, the aircraft} \\ \text{state will require at} \\ \text{least one} \\ \text{maintenance at an} \\ \text{intermediate airport.}\end{array}\right\}$$

According to the methods presented, the probabilities of such events are determined by the formulas:

$$P(A_0) = \sum_{l=1}^{L} P_l (1 - P_{10} - P_{20} - P_{30})^{N_{l-1}} \tag{2.68}$$

$$P(A_1) = \sum_{l=1}^{L} P_l (1 - P_{20} - P_{30})^{N_{l-1}} \tag{2.69}$$

$$P(A_2) = \sum_{l=1}^{L} P_l (1 - P_{30})^{N_{l-1}} \tag{2.70}$$

$$P(\bar{A}_2) = 1 - P(A_2) \tag{2.71}$$

For airports of type "B," the events that determine the dynamics of departure delay formation are the aircraft states from $S = \{S_0, S_1, S_2, S_3\}$ that occur at the end of the flight upon arrival at the base airport.

$$B_0 = B_0(r) = \left[\begin{array}{c} \text{At the end of the } r\text{-th} \\ \text{flight, the aircraft state will} \\ \text{be } S_0. \end{array} \right]$$

$$B_1 = B_1(r) = \left[\begin{array}{c} \text{At the end of the } r\text{-th} \\ \text{flight, the aircraft state will} \\ \text{be } S_1. \end{array} \right] \tag{2.72}$$

$$B_{01} = B_{01}(r) = \left[\begin{array}{c} \text{At the end of the } r\text{-th} \\ \text{flight, the aircraft state will} \\ \text{be } S_0 \text{ or } S_1. \end{array} \right] = \left[\begin{array}{c} \text{Upon arrival at the base} \\ \text{airport at the end of the } r\text{-} \\ \text{th flight, no aircraft} \\ \text{restoration will be} \\ \text{required.} \end{array} \right]$$

At the same time, the events B_0, B_1 and B_{01} satisfy the relation:

$$B_{01} = B \cup B_1, B_0 \cap B_1 = \emptyset \tag{2.73}$$

The last equality means that the events B_0 and B_1 are mutually exclusive.
Let us define the events related to the need for restoration at the base airport:

$$B_2 = B_2(r) = \left[\begin{array}{c}\text{At the end of the } r\text{-th flight, the} \\ \text{aircraft state will be } S_0.\end{array}\right] = \left[\begin{array}{c}\text{Upon arrival at the base airport} \\ \text{at the end of the } r\text{-th flight, no} \\ \text{aircraft restoration will be} \\ \text{required}\end{array}\right]$$

$$B_3 = B_3(r) = \left[\begin{array}{c}\text{At the end of the } r\text{-th flight, the} \\ \text{aircraft state will be } S_1.\end{array}\right] = \left[\begin{array}{c}\text{Upon arrival at the base airport} \\ \text{at the end of the } r\text{-th flight, no} \\ \text{aircraft restoration will be} \\ \text{required.}\end{array}\right]$$

$$B_{23} = B_{23}(r) = \left[\begin{array}{c}\text{At the end of the } r\text{-th flight, the} \\ \text{aircraft state will be } S_0 \text{ or } S_1.\end{array}\right] = \left[\begin{array}{c}\text{Upon arrival at the base airport} \\ \text{at the end of the } r\text{-th flight, no} \\ \text{aircraft restoration will be} \\ \text{required.}\end{array}\right]$$

In this case:

$$B_{23} = B_2 \cup B_3 \tag{2.74}$$

The sequence of random variables $\{æ_r\}$, considered in section 2.2, precisely describes the probabilities of the aircraft states or the probabilities of the listed events. At the same time, the flight type determined by the number l must be taken into account. More precisely, the flight type is defined by the sequence of random variables θ_r.

Let $æ_r^{(l)}$ denote the state of the aircraft at the end of the r-th flight, given that the flight itself had number l. The notation for the corresponding probabilities is expressed as:

$$\rho_0^{(l)}(r) = P\left\{æ_r^{(l)} = \frac{S_0}{\theta_r} = l\right\};$$
$$\rho_1^{(l)}(r) = P\left\{æ_r^{(l)} = \frac{S_1}{\theta_r} = l\right\};$$
$$\rho_2^{(l)}(r) = P\left\{æ_r^{(l)} = \frac{S_2}{\theta_r} = l\right\};$$
$$\rho_3^{(l)}(r) = P\left\{æ_r^{(l)} = \frac{S_3}{\theta_r} = l\right\};$$
$$P\{\theta_r = l\} = P_l, l = 1,2, \dots, L$$

According to the law of total probability, the following formulas are used to find the probabilities of the listed events:

$$P\{B_0\} = P\{B_0(r)\} = \sum_{l=1}^{L} P_l (\rho_0)^{(l)}\,(r) \tag{2.75}$$

$$P\{B_1\} = P\{B_1(r)\} = \sum_{l=1}^{L} P_l(\rho_1)^{(l)}\,(r) \tag{2.76}$$

$$P\{B_2\} = P\{B_2(r)\} = \sum_{l=1}^{L} P_l(\rho_2)^{(l)}\,(r) \tag{2.77}$$

$$P\{B_3\} = P\{B_3(r)\} = \sum_{l=1}^{L} P_l(\rho_3)^{(l)}\,(r) \tag{2.78}$$

The probabilities of events B_{01} and B_{23} are found using the given probabilities by the formulas:

$$\begin{gathered} P\{B_{01}\} = P\{B_0\} + P\{B_1\} \\ P\{B_{23}\} = P\{B_2\} + P\{B_3\} = 1 - P\{B_{01}\} \end{gathered} \tag{2.79}$$

Now it is possible to calculate the average delay time of the aircraft at the base airport taking into account the looping. According to the above, the average delay time at the base airport and its variance are denoted as MV_r and DV_r. These values should be considered as the conditional expectation and conditional variance of the delay time at the base airport, given that the aircraft indeed requires restoration and given that this aircraft is looped into the next flight. Let us introduce the following notation for the events:

$B = B(r) = \left\{ \right.$ At the base airport, after the *r*-th flight, the aircraft is looped into the next flight and requires restoration. $\left. \right\}$

According to the accepted notation, this event can be represented as:

$$B = \{\Psi_r^{(LB)} = 1\} \cap (\{æ_r = S_2\} \cup \{æ_r = S_3\}) \tag{2.80}$$

Each of the events $\left\{\Psi_r^{(LB)} = 1\right\}$, $\{æ_r = S_2\} \cup \{æ_r = S_3\}$ are assumed to be independent. Therefore, the following formula holds for finding the probability of event B:

$$P\{B\} = P_{LB} \cdot P\{B_{23}\} \tag{2.81}$$

Let V_r denote the delay time of the flight at the base airport, which is a random variable with its numerical characteristics. According to the above, the mean value and variance of V_r are found by the formulas:

$$MV_r = P\{B\} \cdot \bar{\varphi}_B = P_{LB} \cdot P\{B_{23}\} \cdot \bar{\varphi}_B$$
$$DV_r = P\{B\} \cdot \bar{d}_B = P_{LB} \cdot P\{B_{23}\} \cdot d_B \quad (2.82)$$

2.6. Application tool implementation of the flight regularity assessment model

Purpose of the application tool

The FlightSync application tool was developed by the author of this dissertation as part of the conducted research. The purpose of creating the application was to provide a practical implementation of the mathematical model for assessing flight regularity, as proposed in Chapter 2.

The developed application tool (FlightSync) serves as a decision support tool for assessing flight regularity, taking into account aircraft maintenance parameters. The program provides a visual representation of the model described in Sections 2.1 and 2.2, and enables its application to various scenarios of airline network operations. The application is intended for engineering and analytical analysis and can also be used in real-world airline operations for optimizing technical logistics [42]. The approach corresponds to modern decision-support applications in maintenance planning described in [43].

The primary end-users of developed application tool are airline departments involved in pre-flight operational planning and risk assessment. These include the Maintenance Control Center (MCC) or engineering control units responsible for coordinating technical support, spare parts positioning, logistics actions, and turnaround buffers. The application tool can also be used by operations control or network planning departments to assess the risk of technical delays across a planned route sequence. In addition, it may support commercial or charter planning departments when evaluating operational risks before confirming flight operations. In practice, FlightSync is used prior to committing to a specific flight or route operation in order to estimate the expected technical delay for the planned operation.

Installation and launch of the program

The program is developed in Python and can be run on any Windows 10/11 system with Python interpreter version 3.10 or higher installed.

To launch the program, follow these steps:

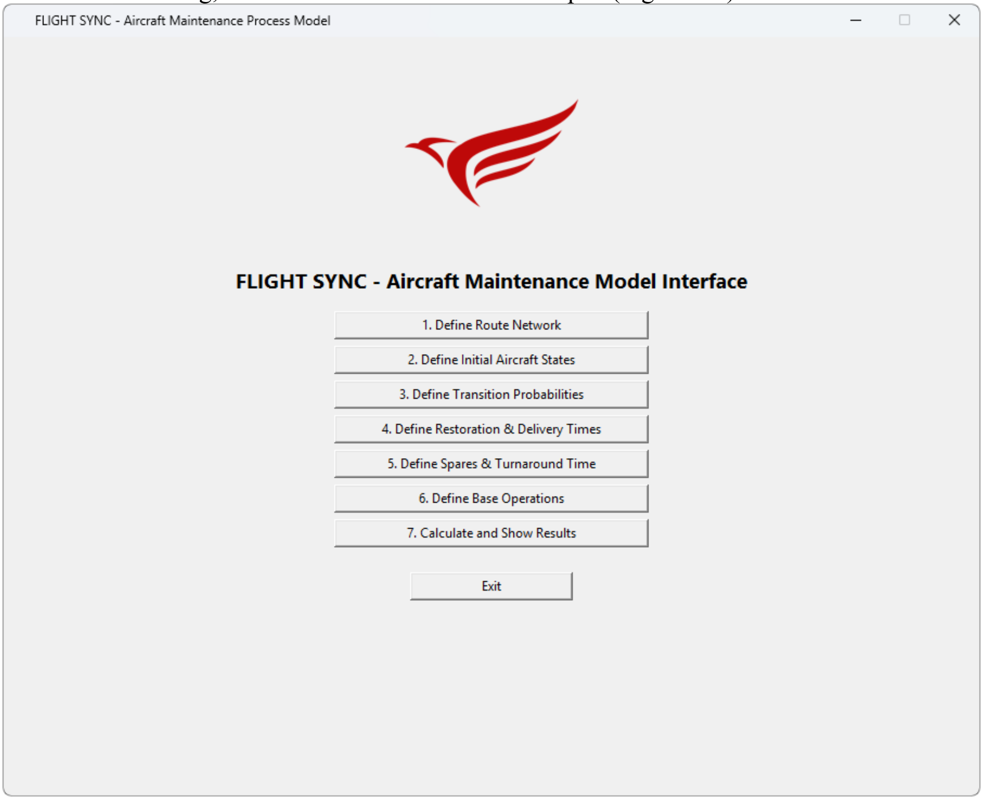
1. Download and install Python from the official website: https://www.python.org/. During installation, make sure to check the box "Add Python to PATH"..
2. Open the Windows Command Prompt (Win+R → type "cmd") and execute the following commands one by one:
   - pip install matplotlib
   - pip install numpy
   - pip install tkinter
   - pip install pillow

If the pip command is not recognized, restart your computer or manually add the path to pip.exe to your system environment variables.

3. Open the standard Windows Notepad. Paste into it the source code provided in Appendix A of this dissertation.
4. Save the file under the name FlightSync.py (File type: *"All Files"*) in the folder C:\FlightSync (create the folder manually if it does not exist).
5. Open the standard Windows Notepad again. Paste the following line into the editor: python FlightSync.py and save the file under the name run_flightsync.bat in the

same folder where FlightSync.py is located. Launch the program by double-clicking the file run_flightsync.bat.

After launching, the main interface window will open (Figure 2.4).



2.4. fig. Main Interface Window of the FlightSync Application Tool.

User interface structure

The model and the program interface utilize three types of airports:

- B – Base airport, where the main technical personnel and spare aircraft are stationed. Example: Riga Airport;
- TB – Transit-base airport, regularly served, with limited recovery capabilities. Example: Vilnius Airport;
- T – Transit airport with minimal technical infrastructure. Example: Tallinn Airport.

The program is implemented as a step-by-step wizard-style interface consisting of seven main screens (steps), each responsible for inputting or analyzing specific parameters of the model. Screenshots and explanations for each step are provided below.

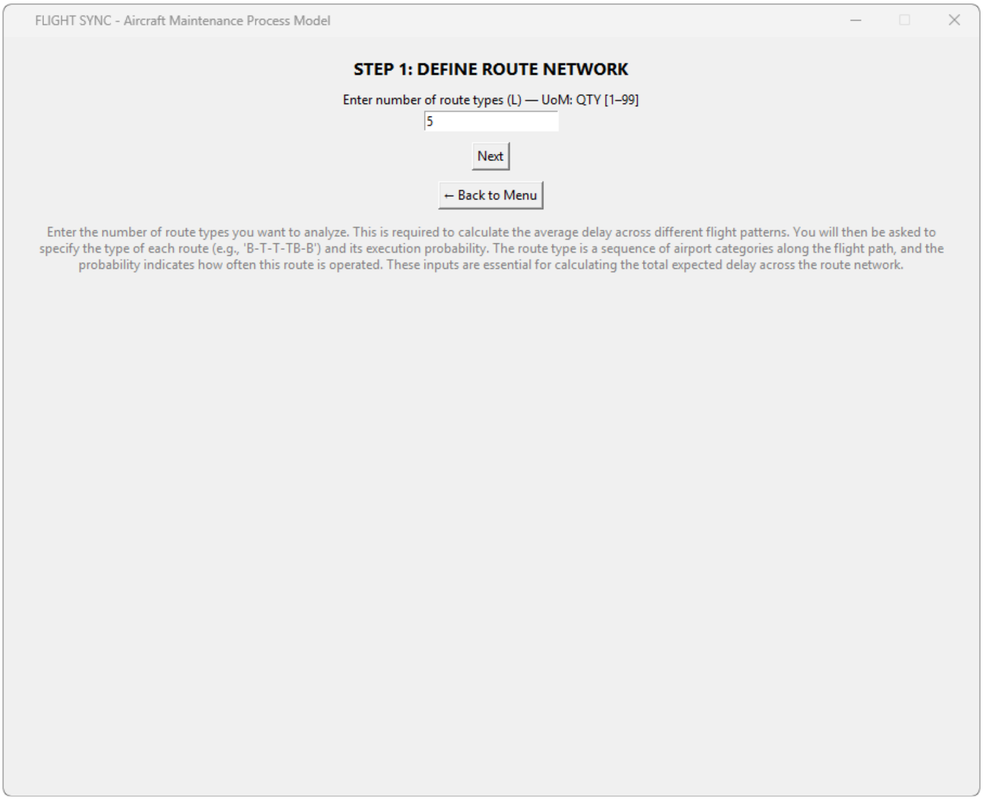
Step 1. Defining route length and flight type probabilities

On the first screen, the user specifies the flight length (number of segments L, e.g., 5) and sets the probabilities for various route configurations.

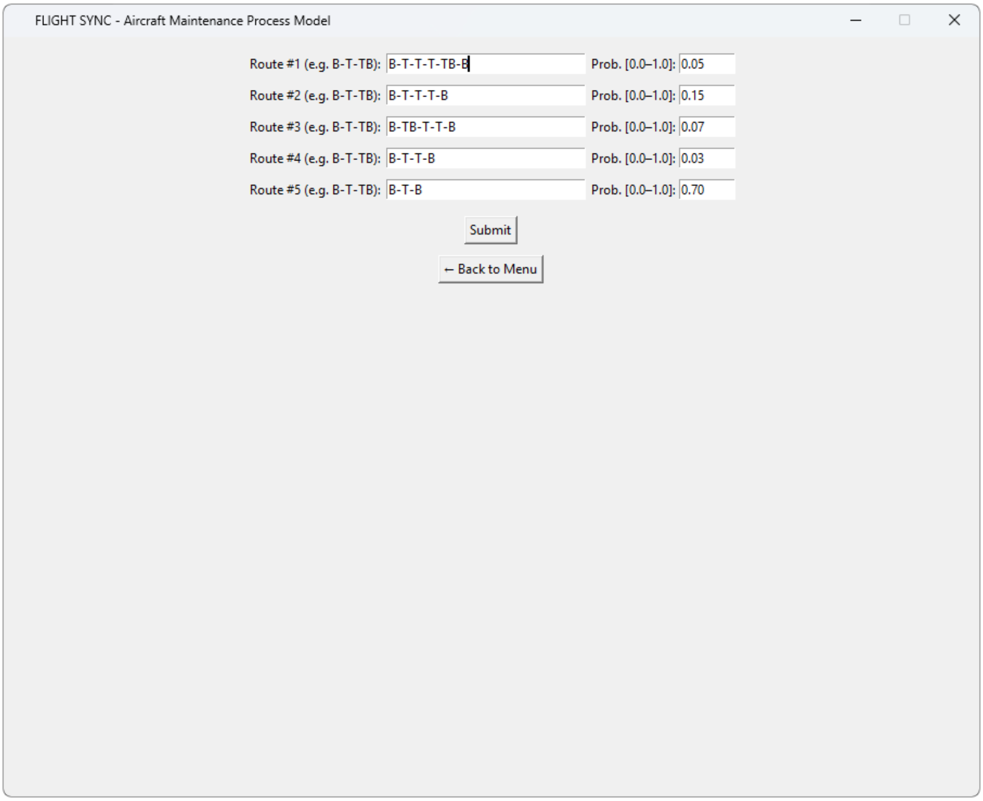
Each route is defined as a sequence of airport types (e.g., B-T-B, B-T-TB-T-B) along with its corresponding probability (Figures 2.5 and 2.6).

The program allows the input of routes with arbitrary probability values; however, it is recommended that the total sum of all probabilities equals 1.0 for accurate route analysis and weighted delay calculations. If needed, the user can manually verify and adjust the total probability.

Note: Route probability represents the share of flights in the overall network that follow a given route. For example, B-T-T-B (p = 0.25) means that 25% of all flights operate along this path.



2.5. fig. Input of Flight Length and Route Network Structure.



2.6. fig. Route Network Flight Structures and Their Probabilities.

Step 2. Defining initial technical states of aircraft

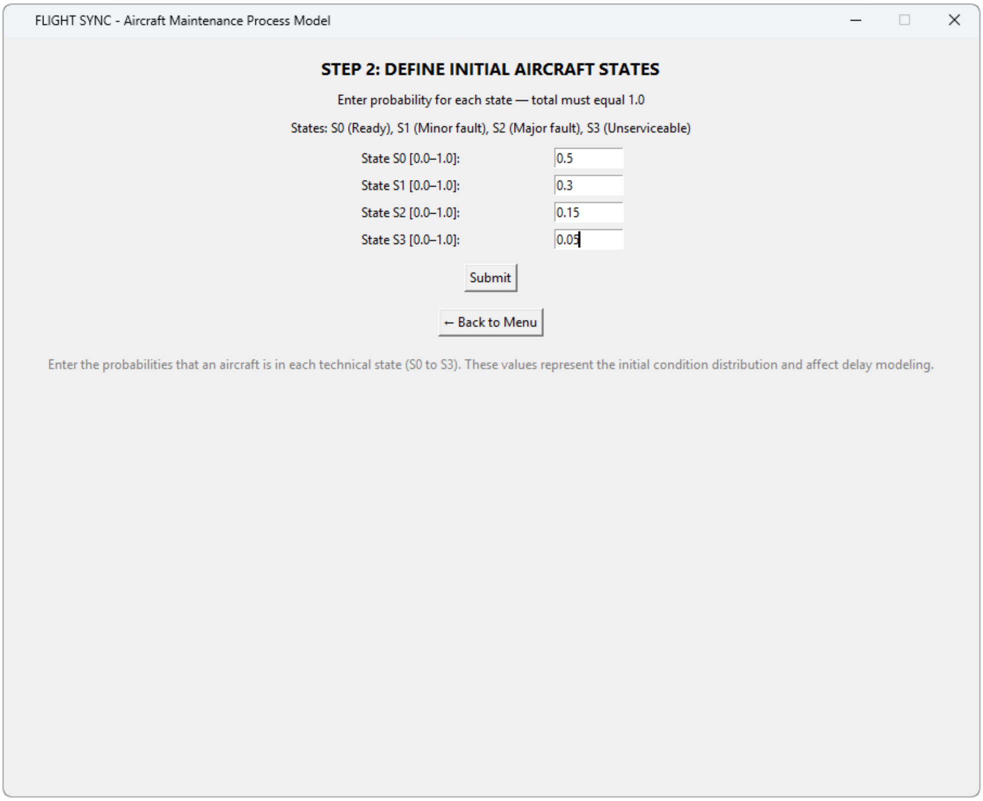
On the second screen (Figure 2.7), the user inputs the probabilities of an aircraft being in one of four technical states (S_0–S_3), representing the condition distribution of the airline's fleet at the time of analysis:

- S_0 – Fully operational condition;
- S_1 – Minor malfunction, dispatch permitted;
- S_2 – Maintenance required;
- S_3 – Restoration required.

Example values:

- S_0: 0.5;
- S_1: 0.3;
- S_2: 0.15;
- S_3: 0.05.

The sum of all values must equal 1.0.



2.7. fig. Input of Initial Aircraft States.

Step 3. State transition matrix

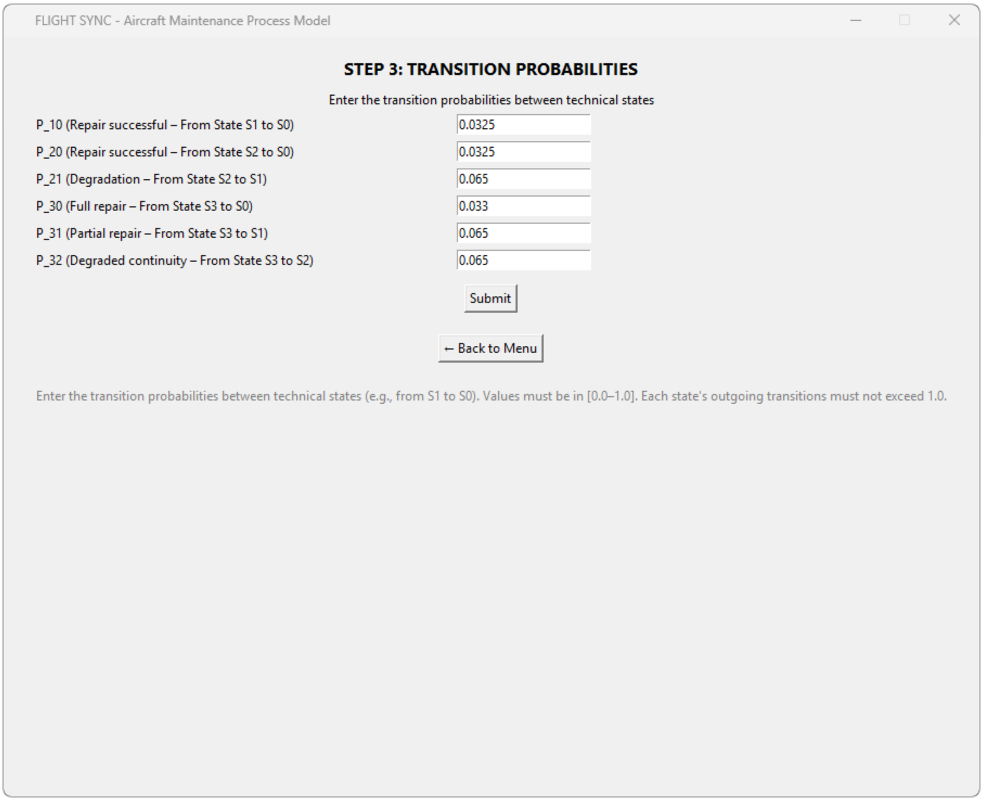
At this step, the user defines the probabilities of transitions between aircraft states after a single flight (Figure 2.8).

Only non-zero elements of the transition matrix P_{ij} are entered:

- P_{10}, P_{20}, P_{30} – transition from state S0;
- P_{21}, P_{31} – transition from state S1;
- P_{32} – transition from state S2.

Example values:

- $P_{10} = 0.0325$;
- $P_{20} = 0.0325$;
- $P_{30} = 0.065$;
- $P_{21} = 0.033$;
- $P_{31} = 0.065$;
- $P_{32} = 0.065$.



2.8. fig. Input of State Transition Probability Matrix.

Step 4. Average restoration time and average delivery time

At this step, the user defines the average restoration time for unserviceable aircraft depending on the airport type. If a spare part is not available on-site, the program calculates additional delivery time required (Figure 2.9):

Example:

Average aircraft restoration time:

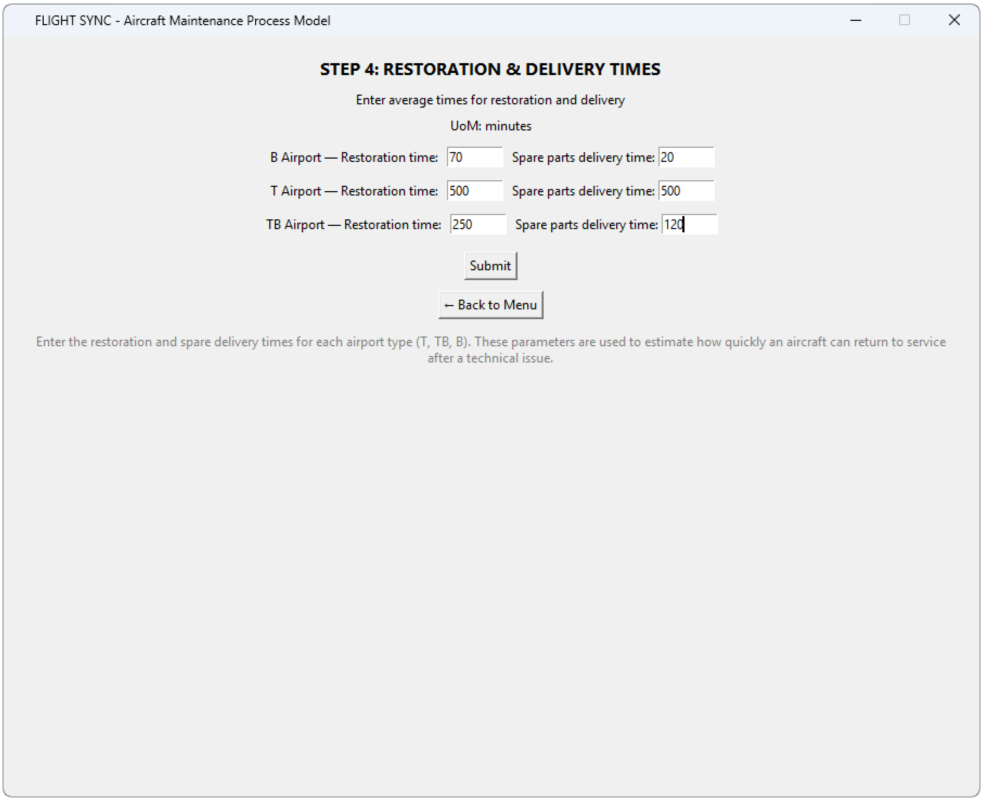
- T (Transit Airport): 500 hours;
- TB (Transit-Base Airport): 250 hours;
- B (Base Airport): 70 hours.

Average spare parts delivery time:

- T (Transit Airport): 500 hours;
- TB (Transit-Base Airport): 120 hours;
- B (Base Airport): 20 hours.

The restoration time accounts for two scenarios: when the spare part is available on-site and when delivery is required. The program calculates the corresponding weighted average using formulas (2.13) and (2.14).

The resulting value is then compared with the turnaround time to determine the delay magnitude at the respective airport.

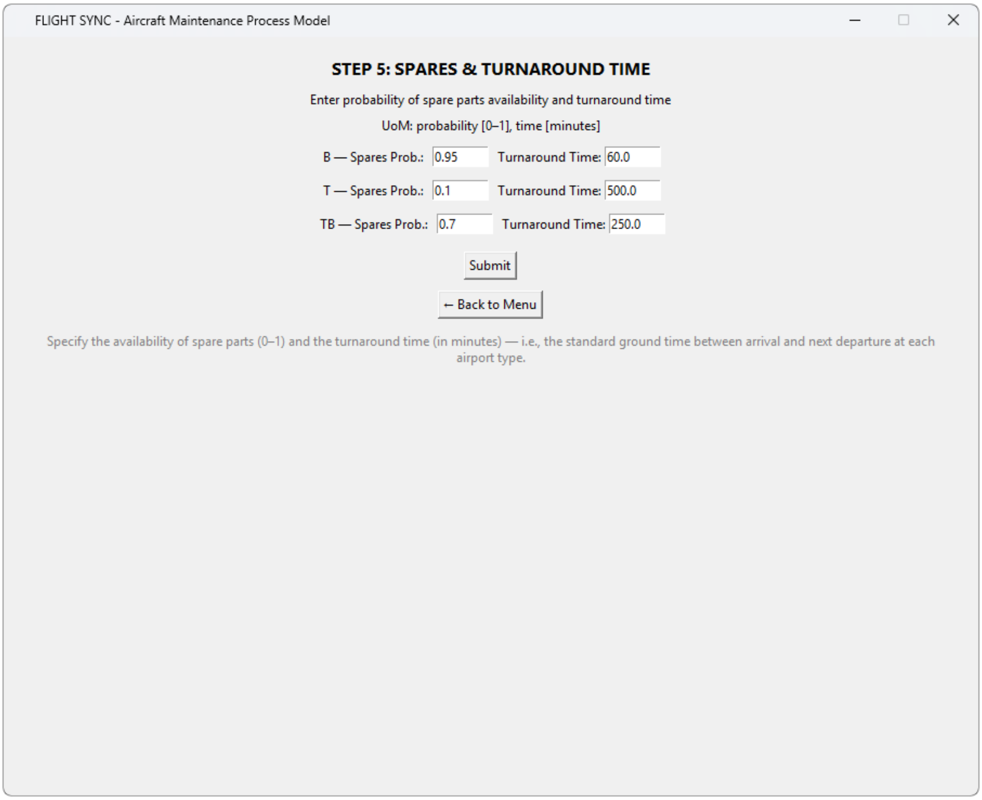


2.9. fig. Input of Average Aircraft Restoration Time and Spare Parts Delivery Time.

Step 5. Spare part availability probability and turnaround time

The user sets the probability that the required spare part is available on-site (P_{SP}) and the standard Turnaround Time for each airport type (Figure 2.10):
Example:

- T (Transit Airport): P_{SP}^{T} = 0.1, Turnaround = 1000 minutes
- TB (Transit-Base Airport): P_{SP}^{TB} = 0.7, Turnaround = 250 minutes
- B (Base Airport): P_{SP}^{B} = 0.95, Turnaround = 60 minutes



2.10. fig. Input of Spare Part Availability Probabilities and Turnaround Time.

Step 6. Base airport parameters

At this step, the user defines parameters related to the possibility of returning the aircraft to the base and the subsequent use of a spare aircraft (Figure 2.11).

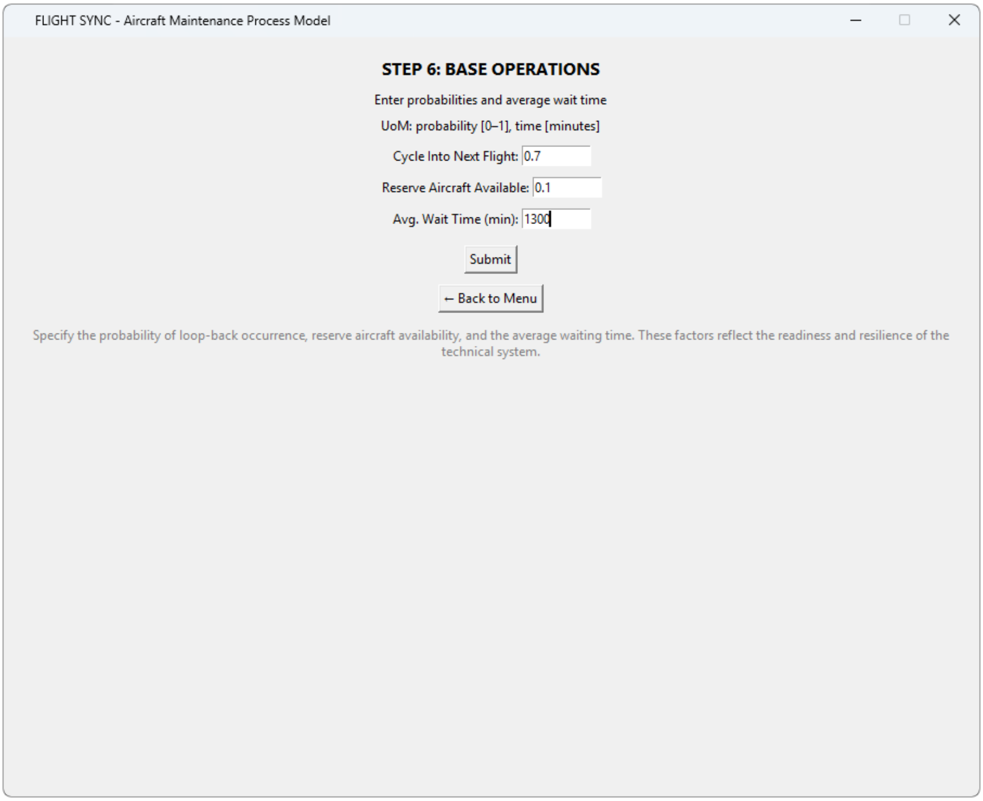
The probability of return to base is denoted as P_{LB}. If a return occurs, the program calculates the delay associated with the unavailability of a spare aircraft.

If no spare aircraft is available (with probability $1 - P_{RES}$), an additional delay equal to T_{RES} is added to the final flight regularity calculation.

This logic allows for modeling real-world operational constraints, such as delays due to spare part availability, maintenance queueing, and resource limitations [44].

Example:

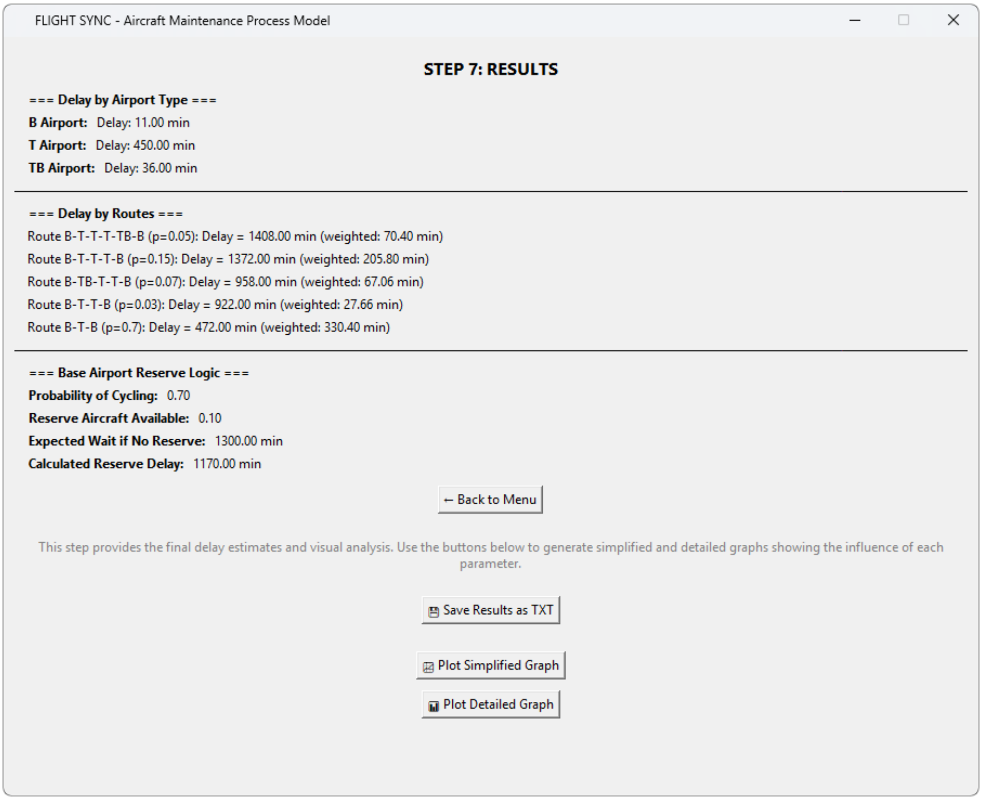
- P_{LB} = 0.7;
- P_{RES} = 0.1;
- P_{RES} = 1300 min.



2.11. fig. Input of Base Airport Parameters.

Step 7. Calculation and result analysis
On the final screen (Figure 2.12), the following are displayed:

- Calculated delay values for each airport type;
- Route-specific delays weighted by their probabilities;
- Calculation related to spare aircraft usage;
- Buttons for generating simplified and detailed delay charts;
- Button to save all input and output data in ".txt" format.



2.12. fig. Delay Calculation and Analysis.

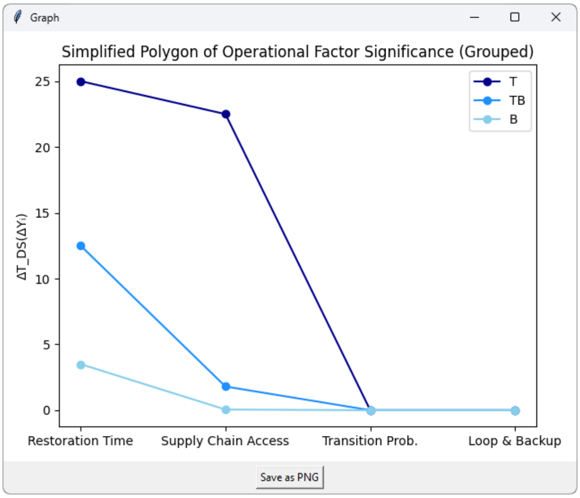
For each route, the program calculates the delay and then computes its weighted value based on the user-defined probability. This approach allows us to consider the contribution of each route to the overall flight regularity.

Saving the results

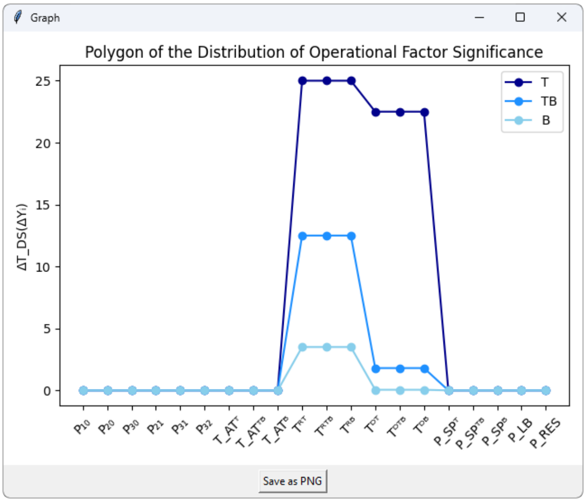
The program allows saving all input data and calculated results as both charts (Figures 2.13 and 2.14) and a text file. To do this, simply click the "Save Results as TXT" button on Step 7. The following data are saved:

- All user-defined parameters;
- Calculated delays for each airport type;
- Delays per route;
- Results of the spare aircraft logic calculation.

The file is convenient for use as an appendix to a report, a presentation, or for further analytical processing.



2.13. fig. Simplified Chart.



2.14. fig. Detailed Chart.

Connection to the mathematical model

Each step of the interface corresponds to specific elements of the model described in Sections 2.1 and 2.2. For instance, the initial states and transition matrix implement the scheme of technical condition degradation. Steps 3–7 reflect the structure of restoration and logistics. The calculations in Step 8 correspond to formula (2.14), while the sensitivity charts implement the analysis presented in Chapter 3 of this dissertation. Thus, the FlightSync program serves as a complete and practical implementation of the proposed mathematical model and can be used as an engineering decision support tool.

Comparative analysis of existing models of delay formation

In the field of operational flight regularity analysis, various mathematical approaches have been proposed to assess delays caused by technical and logistical disruptions. These include classical queueing models, stochastic delay simulations, and reliability-centered formulations. However, most of these models exhibit significant limitations when applied to real-world operational decision-making, especially at the airline level.

Table 2.1 summarizes the key characteristics of major modeling approaches found in the literature and compares them with the approach developed in this dissertation. The table highlights the distinct features of the proposed method, particularly in its integration of dual-level MEL logic, differentiated airport recovery capabilities, probabilistic part delivery times, and operational control logic such as reserve aircraft availability and loop-back behavior.

Unlike traditional models that often treat airport recovery conditions as uniform or assume deterministic restoration times, the proposed model introduces a multi-dimensional probabilistic framework that explicitly accounts for structural, technical, and organizational variables influencing flight delays. Moreover, the inclusion of a dedicated application tool (FlightSync) distinguishes this study by enabling scenario-driven, data-supported analysis of delay dynamics.

The comparison below emphasizes these advances in terms of modeling detail, realism, and operational usability.

Table 2.1

Comparison of Flight Delay Modeling Approaches

Model / Approach	MEL Consideration	Airport Differentiation	Spare Aircraft Logic	Probabilistic Restoration Modeling	Software Implementation
Classical Queueing Models	Not included	Homogeneous assumptions	Not modeled	Average-based only	Absent
Stochastic Delay Simulations	Simplified or ignored	Partial (hub vs. spoke)	Rarely included	Often deterministic	Limited prototyping
Reliability-Focused Models	Basic dispatch constraints	No differentiation	Not modeled	Failure probabilities only	Absent or theoretical
This Study (FlightSync Model)	Dual-level MEL modeling	Full: B, T, TB structure	Reserve + loopback	Fully probabilistic with delivery	Dedicated software tool

Table 2.1 is developed by the author based on comparative analysis of representative academic and industrial publications, including [17], [18], [30], [32], [34], [40], and [43] in the bibliography.

Limitations and scope of the proposed model

While the developed model provides a detailed and practical framework for analyzing aircraft maintenance-related delays, it is important to acknowledge its assumptions and boundaries. These limitations are inherent to the modeling approach and stem from the need to balance analytical tractability with operational realism.

The following aspects define the scope and constraints of the proposed methodology:

- Stationary failure rates - the model assumes that aircraft failure rates remain constant during the period under analysis. This simplification ignores long-term aging effects or variations due to environmental or seasonal conditions. In reality, the reliability of systems may degrade over time, especially if condition-based monitoring is not integrated.

- Homogeneous aircraft fleet behavior - all aircraft are treated as statistically identical in terms of failure probability and maintenance characteristics. Differences in age, configuration, or technical status are not modeled individually. This assumption is justified for short-term operational planning but may require refinement in heterogeneous fleets.
- Discrete operational states and transitions - the model defines aircraft condition using a limited set of discrete technical states (S_0–S_4), with fixed transition probabilities. While this provides a clear structure, it abstracts away continuous degradation processes or compound failure scenarios.
- No explicit modeling of crew- or ground-induced delays - the model focuses exclusively on delays caused by aircraft technical readiness and associated logistics. It does not account for delays resulting from crew unavailability, ground service issues, or airport congestion though such delays may interact with technical disruptions in practice.
- Simplified representation of maintenance logistics – spare part delivery and restoration processes are modeled probabilistically using average time values and availability probabilities. The model does not capture queueing effects, parallel maintenance operations, or delays caused by external logistics providers [53], which may affect actual turnaround performance.
- Assumed independence of route assignments - the model treats the assignment of aircraft to route types as a stochastic process with known probabilities. It does not dynamically optimize fleet routing or incorporate operational rescheduling, although these actions may influence delay propagation in practice.

Despite these limitations, the proposed model effectively captures the essential relationships between aircraft state dynamics, airport-specific recovery capabilities, and delay formation. It provides a robust foundation for short-term operational decision-making and offers a basis for future expansion, including integration with dynamic scheduling tools or more granular reliability models.

Statistical data considerations and operational factors influencing model parameters

The reliability of probability estimates used in the proposed model depends on several factors, including the frequency of observed technical events, the stability of operational conditions, and the duration of the observation period. For probabilistic models describing aircraft technical state transitions, the statistical dataset must contain a sufficient number of operational observations in order to ensure representative estimation of transition probabilities between aircraft technical states.

In general, the use of multi-year datasets is recommended in order to capture seasonal variability of airline operations and to reduce sensitivity to short-term operational anomalies. Seasonal operational factors may include winter operations, de-icing procedures, weather-related disruptions, and variations in logistics performance or airport congestion. A sufficiently long observation period also increases the probability of capturing rare but operationally significant events, such as AOG-critical transitions, which are essential for estimating low-probability transitions within the model.

In this research, the dataset used for calibration of the model covers approximately 2 years and 8 months of operational observations (March 2023 – November 2025) and includes 40 recorded technical events. During this period the aircraft fleet maintained an average utilization of approximately 110 flight hours per month and approximately 48 flight legs per month, providing substantial operational exposure for observing aircraft state transitions. Considering both the duration of the observation period and the operational intensity of aircraft utilization, the dataset provides a sufficiently representative empirical basis for estimating transition probabilities within the proposed modelling framework. Therefore, the available statistical data were considered adequate for the intended objective of the study, namely calibration of the model parameters and practical validation of the expected technical delay estimation within the case-study environment.

In practical airline operations, the statistical parameters of the model should not be considered static. Aircraft technical condition, maintenance capability, logistics performance, and operational procedures may evolve over time. Therefore, periodic updating of model parameters is recommended as new operational data becomes available. A rolling update policy may be applied, where state transition statistics are updated when new technical events are recorded or according to a predefined schedule (for example monthly or quarterly) based on operational sources such as defect logs, Maintenance Control Center records, and structured maintenance documentation. Similarly, logistics-related parameters such as spare parts availability and delivery time should be periodically reassessed using operational logistics performance data.

For initial calibration of the model, multi-year operational datasets can serve as a baseline. However, for maintaining operational relevance of probability estimates, it is recommended to use a rolling observation window covering up to the most recent five years of operational data. Such an approach allows the model parameters to reflect the current technical condition of the fleet, the performance of the logistics system, and the capability of the maintenance organization.

In probabilistic modelling, the use of confidence intervals and other statistical reliability measures may improve the robustness of probability estimates, particularly when analyzing rare events. However, the primary objective of the present research was to develop an applied operational model based on airline-level operational statistics and to demonstrate its practical use through scenario analysis and case-study validation. The introduction of formal confidence intervals would require detailed event counts for each transition, additional statistical assumptions regarding the distribution and independence of events, and more granular component-level reliability data. Such extensions may be considered in future developments of the modelling approach, including the use of confidence bounds or Bayesian updating procedures for low-frequency events.

Operational factors such as seasonal variations may also influence aircraft recovery capability and, consequently, the probability and duration of departure delays. Seasonal effects may include winter operations, de-icing procedures, weather-related disruptions, airport congestion, and variations in ground handling performance. These factors may be incorporated into the modelling framework by estimating separate parameter sets for different operational periods (for example winter and summer seasons) or by introducing time-dependent coefficients for recovery and logistics processes. In the present research these factors were not explicitly modelled in order to maintain the tractability of the mathematical model and to focus on the core mechanism of technical delay formation driven by aircraft technical state transitions and maintenance and logistics capability. Nevertheless, the proposed modelling framework allows seasonal stratification of statistical parameters as a potential extension.

The developed mathematical model estimates expected technical delay based on aircraft technical state transitions and recovery capability at network airports. Therefore, the model itself is independent of the commercial type of flight operation and may be applied to both scheduled and charter flights. Operational differences between these types of operations, such as turnaround buffer policies, dispatch flexibility, or commercial constraints, were not explicitly modelled in this study because the primary objective was to analyze the technical mechanisms of delay formation. However, such operational differences may be incorporated in future developments of the modelling framework through the introduction of additional parameters or operational profiles representing different flight operation types.

Similarly, the model does not explicitly distinguish between passenger and cargo aircraft configurations as separate operational modes. The modelling approach is based on aircraft technical state transitions and recovery capability and therefore focuses primarily on the technical condition of the aircraft and the maintenance environment. Nevertheless, configuration-related differences may influence operational constraints and dispatch conditions. For example, the relevance of cabin systems, emergency equipment requirements, or dispatch limitations defined by the Minimum Equipment List may differ between passenger and cargo aircraft. These differences were not explicitly modelled in the present study but may be incorporated in future extensions of the modelling framework through the introduction of configuration-specific parameter sets.

Overall, the proposed modelling approach provides a flexible framework for analyzing technical delay formation in airline operations. Additional operational factors and statistical refinements may be incorporated through parameter extensions without requiring fundamental changes to the structure of the mathematical model.

Dual-level MEL classification and dispatch logic

In real-world airline operations, the Minimum Equipment List (MEL) serves as a regulatory mechanism that allows aircraft to be dispatched with certain inoperative components under controlled conditions. However, not all failures have equal operational consequences, particularly when considering differences in airport recovery capabilities and network structure. To reflect this, the proposed model introduces a dual-level MEL classification system that differentiates between two operationally distinct categories of allowable failures:

- Level 1 – "To Base" allowances - this category includes failures that allow the aircraft to continue operating only until it reaches a certified base airport (B) with sufficient maintenance capability. These items typically affect systems that, while not critical for immediate safety, may compromise reliability if not addressed promptly. Under this classification, the model restricts further dispatch from non-base airports unless recovery occurs;
- Level 2 – "To Next Maintenance" allowances - failures in this category are considered sufficiently minor to be deferred until the next scheduled maintenance event, even if the aircraft passes through transit (T) or transit-base (TB) airports. These include items whose failure does not significantly impact aircraft performance, safety, or dispatch reliability, and which are permissible under the MEL for extended operation.

This two-tiered classification is integrated into the transition logic of the model. Specifically, after a degradation event (e.g., transition from state S_0 to S_1), the type of failure determines whether the aircraft:

- Must be routed to a base airport for restoration (if it falls under Level 1);
- May continue operating on its planned route (if allowed under Level 2).

The model assigns a probability of unjustified failure tolerance, which represents the risk of applying Level 2 allowances inappropriately i.e., assigning "to next maintenance" status to failures that should require base restoration. This introduces a realistic mechanism for evaluating the consequences of MEL misuse or risk-based decision errors.

By explicitly modeling these two MEL levels, the framework enables analysis of how technical policy, risk tolerance, and dispatch strategy influence delay formation. It also allows for simulation of different MEL configurations and their impact on operational reliability across various airport types.

2.7. Comparative Analysis of Existing OCC Systems

Operations Control Centre (OCC) systems are designed to support flight coordination, schedule planning, monitoring of technical status and resources, and handling deviations from the original flight schedule. OCC systems are widely adopted by both passenger and cargo airlines.

The most commonly used commercial solutions in this domain include Lufthansa NetLine/Ops, Sabre AirCentre Movement Manager, Jeppesen Ops Control (by Boeing), and SITA Horizon. These systems provide tools for visualizing flights, fleet management, slot scheduling, routine maintenance tracking, and crew assignment.

However, a comparative analysis of their functionality, based on available public documentation, technical specifications, and user manuals, reveals that the listed systems do not offer capabilities for simulation-based modeling of technical delays, probabilistic modeling of aircraft technical states, or logistical dependencies such as spare part delivery times, airport classification (B, T, TB), or reserve aircraft allocation. These factors are often addressed manually by operations personnel or handled externally via disconnected systems.

The following table summarizes the key differences between standard OCC platforms and the proposed model presented in Table 2.2:

Table 2.2

Comparison of FlightSync with existing OCC systems

Parameter	**Standard OCC Systems**	**Proposed FlightSync Model**
Scheduling and Slot Management	Yes	Yes
Maintenance Tracking	Yes (fixed intervals)	Yes (dynamic transitions)
Logistics of Spare Parts and Reserves	Limited / Manual	Mathematically Modeled
Probabilistic Aircraft States (S_0–S_3)	No	Yes
Simulation Modeling	No	Yes
Airport Category Consideration (B, T, TB)	No	Yes
Cumulative Delay Calculation by Route	No	Yes

Thus, despite the maturity of modern OCC systems, they do not provide tools for probabilistic assessment of how technical infrastructure and logistical constraints influence flight regularity. The proposed model addresses this gap by utilizing discrete Markov chains, simulation-based modeling, and cumulative delay analysis.

Table 2.2 was developed by the author based on a comparative analysis of representative academic and industrial sources, including references [46], [47] and [48] from the bibliography.

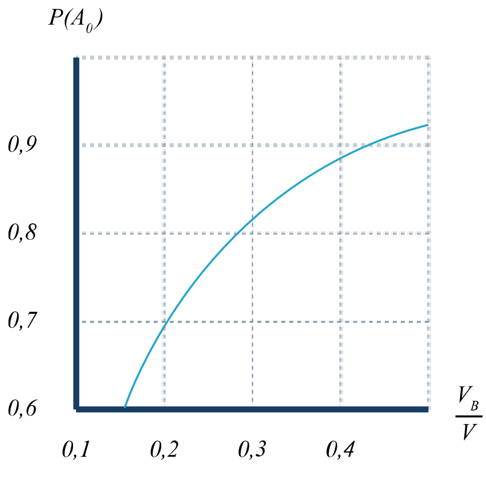
3. DEVELOPMENT AND JUSTIFICATION OF RECOMMENDATIONS FOR IMPROVING THE OPERATIONAL MANAGEMENT OF AIRCRAFT TECHNICAL OPERATION PROCESSES UNDER AIRLINE CONDITIONS

3.1. Analysis of the dynamics of aircraft states at airports within the airline network

The analysis of the dynamics of aircraft states at airports within the airline network is necessary for assessing the number of situations in which aircraft recovery is required, and therefore, the risk of departure delays arises. At the same time, the influence of the Minimum Equipment List (states S_1 and S_2) as a mechanism for managing these dynamics has not been sufficiently studied in research known to the author. Therefore, in the analysis results presented below, this issue is given primary attention.

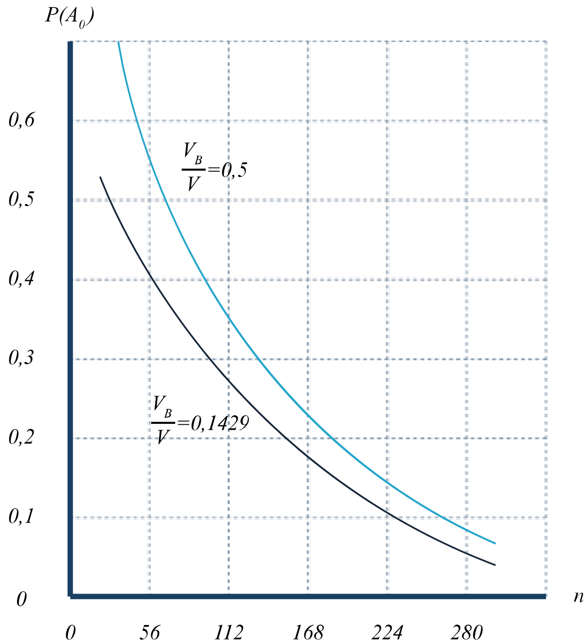
Since there are differences in the processes of aircraft recovery and the formation of departure delays at base and non-base airports, the analysis of the dynamics of aircraft states has been carried out separately for the non-base and base airports within the airline network.

The dependency shown in Figure 3.1 for the probability $P(A_0)$ of completing a flight at non-base airports in state S_0 demonstrates that the absence of a Minimum Equipment List (MEL), as well as having an MEL containing only state S_2, results in the dynamics of $P(A_0)$ being determined solely by the number of landings per flight in non-base airports (parameter $\frac{V_B}{V}$). The inclusion in the MEL of failures that allow continued flights until the next scheduled maintenance check (state S_1) leads (Figure 3.2) to additional dynamics in the probability $P(A_0)$ based on the number of flights performed, n.



$$P_{00} = 0{,}92;\ P_{10} = 0; \frac{P_{20}}{P_{30}} - var; \frac{P_{32}}{P_{30}} - var; n - var$$

3.1.fig. Dynamics of state S_0 at $P_{10} = 0$ in non-base airports.



$$P_{00} = 0{,}92; \frac{P_{10}}{1 - P_{00}} = 0{,}125; \frac{P_{20}}{P_{30}} - var; \frac{P_{21}}{P_{20}} - var; \frac{P_{32}}{P_{30}} = 1; \frac{P_{31}}{P_{30}} = 1$$

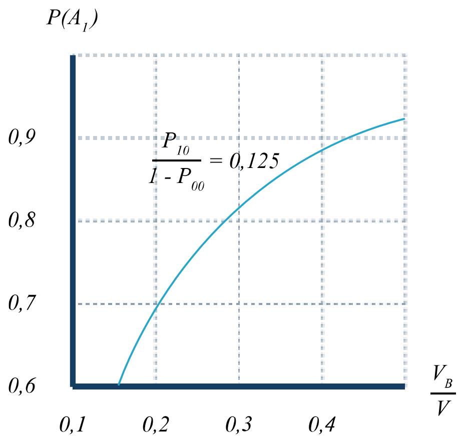
3.2.fig. Dynamics of state S_0 at $P_{10} \neq 0$ in non-base airports.

The indicated nature of the dynamics of state S_0 in non-base airports does not contradict the previously introduced definitions of aircraft states and their regeneration cycles in the process of aircraft technical operations of aircraft.

Considering the dynamics of the probability $P(A_1)$ of flight completion in non-base airports in states S_0 or S_1, as shown in Figure 3.3, it should be noted that it does not change with respect to the parameter n. In this regard, the decrease in the probability $P(A_0)$ with respect to parameter n observed in Figure 3.2 indicates an increase in the probability of the aircraft being in state S_1 as the parameter n increases.

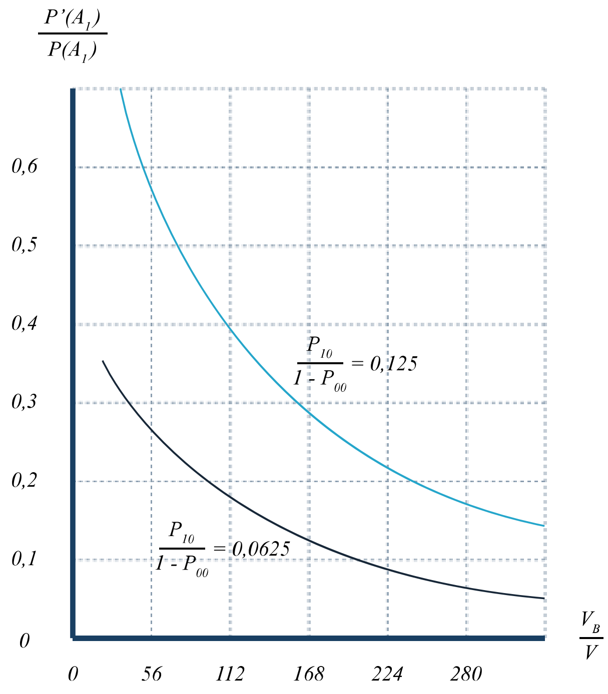
As follows from the dependence shown in Figure 3.4, the expansion of the list of permissible failures by introducing failures that allow continuation of flights until the next scheduled maintenance (expanding the set of aircraft states from $\{S_0, S_2, S_3\}$ to $\{S_0, S_1, S_2, S_3\}$), while maintaining the total failure rate $(1 - P_{00})$ per flight, leads to a slight increase in the probability $P(A_1)$ compared to the probability $P(A_0)$. Moreover, the gradient of this increase becomes greater as the number of landings per flight in non-base airports increases (i.e., as the parameter $\frac{V_B}{V}$ decreases).

The dependencies shown in Figure 3.5 of the probability on the level of justification of the list of permissible failures those allowing continuation of flights until the next scheduled maintenance with respect to failures characteristic of the state (parameter $\frac{P_{21}}{P_{20}}$), at various values of the parameter $\frac{V_B}{V}$, demonstrate the presence of dynamics in the probability $P(A_1)$ with respect to the parameter n, which was not observed previously. This is explained by the aircraft being restored at the base airport after completing the flight.



$$P_{00} = 0{,}92;\ \frac{P_{21}}{P_{20}} = 1; \frac{P_{20}}{P_{30}} - var; n - var; \frac{P_{32}}{P_{30}} - var; \frac{P_{31}}{P_{30}} - var$$

3.3.fig. Dynamics of the state (S_0US_1) in non-base airports.



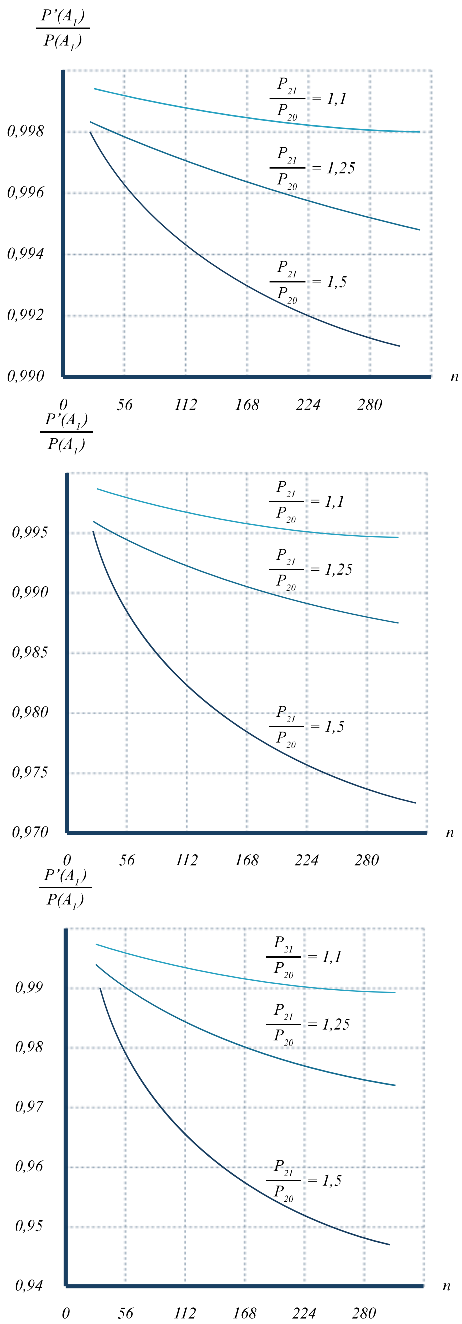
$$P_{00} = 0{,}92;\ \frac{P_{31}}{P_{30}} = 1; \frac{P_{32}}{P_{30}} = 1; \frac{P_{21}}{P_{20}} = 1; \frac{P_{20}}{P_{30}} - var; n - var; P(A_1) \text{ at } P_{10} = 0$$

3.4.fig. Dynamics of changes in the probability of states (S_0US_1) with the expansion of the list of allowable failures by failures permitting continuation of flights until the next scheduled maintenance.

a) $\dfrac{V_B}{V} = 0{,}5$

b) $\dfrac{V_B}{V} = 0{,}3333$

c) $\dfrac{V_B}{V} = 0{,}1429$



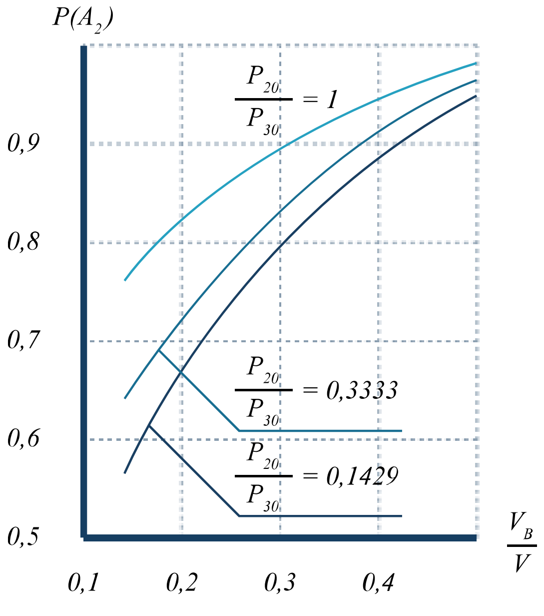
$$P_{00} = 0{,}92;\ \frac{P_{10}}{1 - P_{00}} = 0{,}125;\frac{P_{20}}{P_{30}} = 0{,}3333;\frac{P_{31}}{P_{30}} = 1;\ P(A_1) \text{ at } \frac{P_{21}}{P_{20}} = 1$$

3.5.fig. Dynamics of changes in the probability of states $(S_0 \mathrm{U} S_1)$ at various values of parameters $\frac{P_{21}}{P_{20}}$ and $\frac{V_B}{V}$ in non-base airports.

The inadequacy of the allowable failures list with respect to the parameter $\frac{P_{21}}{P_{20}}$ leads to a decrease in the probability $P(A_1)$, which increases with the growth of the parameter n and the number of landings per flight in non-base airports (decrease in the parameter $\frac{V_B}{V}$).

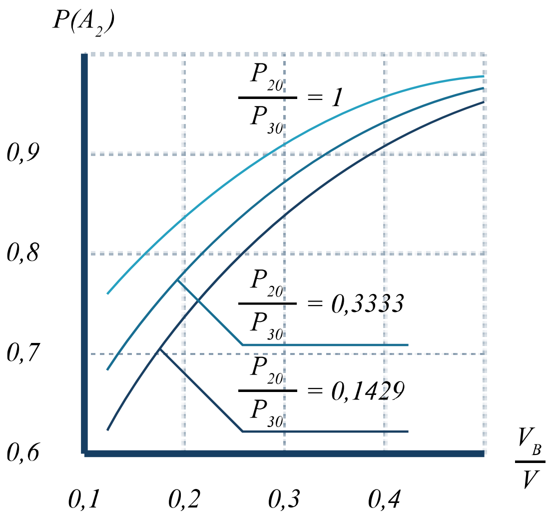
Comparing the dependencies in Figure 3.5 and Figure 3.4, it should be noted that the impact of the validity level of the allowable failures list with respect to the parameter $\frac{P_{21}}{P_{20}}$ on the value of $P(A_1)$ is less significant compared to the procedure of expanding the set of aircraft states from $\{S_0, S_2, S_3\}$ to $\{S_0, S_1, S_2, S_3\}$ (which increases the parameter $\frac{P_{10}}{1-P_{00}}$).

As follows from the dependencies shown in Figure 3.6 and Figure 3.7, the dynamics of the probability $P(A_2)$ of completing a flight at non-base airports in states S_0, S_1, or S_2 is observed only with respect to the parameter $\frac{V_B}{V}$ and is similar in nature to the dynamics of the probability $P(A_1)$. At the same time, the probability $P(A_2)$ does not depend on the parameter $\frac{P_{21}}{P_{20}}$ and increases with a decrease in the number of landings per flight at non-base airports (increase in the parameter $\frac{V_B}{V}$), as well as with an increase in the proportion of failures allowing continued flight to the base airport (parameter $\frac{P_{20}}{P_{30}}$), and with the expansion of the set of aircraft states from $\{S_0, S_2, S_3\}$ to $\{S_0, S_1, S_2, S_3\}$ (increase in the parameter $\frac{P_{10}}{1-P_{00}}$).



$$P_{00} = 0{,}92;\ P_{10} = 0; \frac{P_{32}}{P_{30}} = 1;\ n - var$$

3.6.fig. Dynamics of changes in the probability of states ($S_0 V S_1 V S_2$) for various values of the parameter $\frac{P_{20}}{P_{30}}$ and with $P_{10} = 0$ at non-base airports.



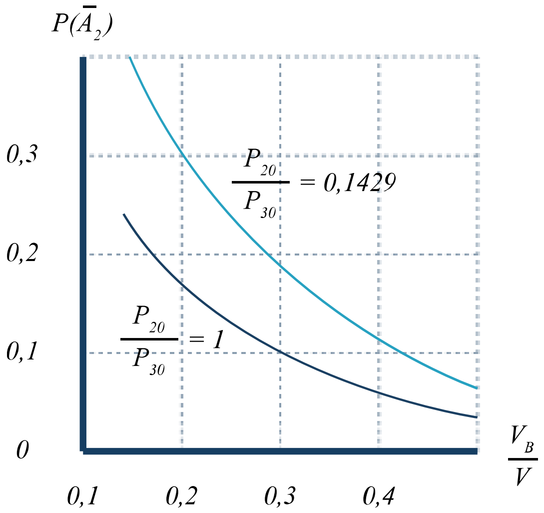
$$P_{00} = 0{,}92; \frac{P_{10}}{1 - P_{00}} = 0{,}125; \frac{P_{21}}{P_{20}} - var; \frac{P_{31}}{P_{30}} = 1; \frac{P_{32}}{P_{30}} = 1; n - var$$

3.7.fig. Dynamics of changes in the probability of states ($S_0 \mathrm{V} S_1 \mathrm{V} S_2$) for various values of the parameter $\frac{P_{20}}{P_{30}}$ and with $\frac{P_{10}}{1-P_{00}} = 0{,}125$ at non-base airports.

Summarizing the results of the analysis of the dependencies presented in Figs. 3.1–3.7, which describe the dynamics of aircraft states that do not require restoration at non-base airports, the following main conclusions can be drawn:

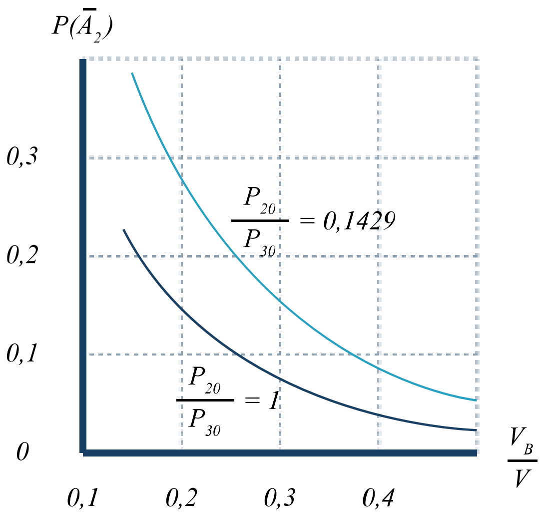
- The airline network, on which the aircraft fleet is operated, consisting of flights with fewer landings at non-base airports (higher values of the parameter $\frac{V_B}{V}$), all other things being equal, provides a higher probability of absence of situations that cause the need for aircraft restoration at these airports;
- The introduction of the list of permissible failures (states S_1 and S_2) generally increases the probability of the absence of situations at non-base airports that require aircraft restoration, i.e., it is considered an effective means of influencing aircraft flight regularity indicators within the airline network;
- The inclusion in the list of permissible failures of those failures that allow continuing the flight until the next scheduled maintenance (due to the increase in the probability $P(A_1)$ and the decrease in the probability $P(A_0)$ as n grows) leads to an increased likelihood of the presence of these failures on the aircraft as its remaining time until the next scheduled maintenance decreases;
- The unjustified list of failures that allow continuing flights until the next scheduled maintenance, in relation to failures characteristic of state S_2 (parameter $\frac{P_{21}}{P_{20}}$), does not affect the probability of the absence of situations requiring aircraft restoration in non-base airports.

The probability of occurrence of situations in non-base airports that require aircraft restoration in these airports is described by the probability $P(\bar{A}_2) = 1 - P(A_2)$. Therefore, the nature of the dependencies shown in Figures 3.8 and 3.9 is completely determined by the dependencies presented in Figures 3.6 and 3.7, respectively. A comparison of the dependencies in Figures 3.8 and 3.9 shows that the expansion of the set of aircraft states from $\{S_0, S_2, S_3\}$ to $\{S_0, S_1, S_2, S_3\}$, while maintaining the overall failure flow $(1 - P_{00})$ per flight, does not lead, under equal conditions, to a significant reduction in the probability $P(\bar{A}_2)$.



$$P_{00} = 0{,}92;\ P_{10} = 0; \frac{P_{32}}{P_{30}} = 1;\ n - var$$

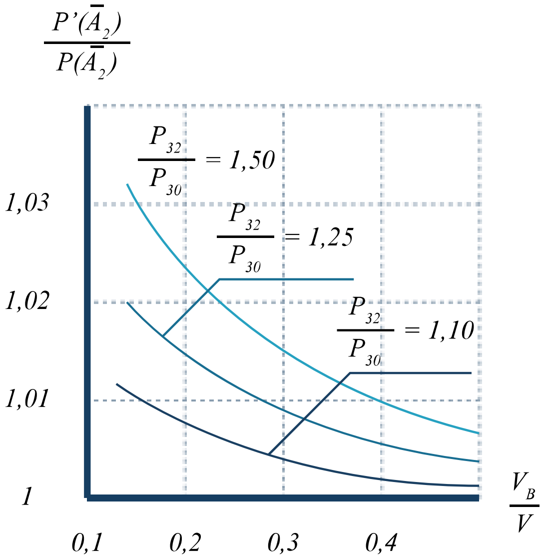
3.8.fig. Dynamics of changes in the probability of state S_3 for various values of the parameter $\frac{P_{20}}{P_{30}}$ and at $P_{10} = 0$ in non-base airports.



$$P_{00} = 0{,}92; \frac{P_{10}}{1 - P_{00}} = 0{,}125; \frac{P_{21}}{P_{20}} = 1; \frac{P_{31}}{P_{30}} = 1; \frac{P_{32}}{P_{30}} = 1; n - var$$

3.9.fig. Dynamics of changes in the probability of state S_3 for various values of the parameter $\frac{P_{20}}{P_{30}}$ and at $\frac{P_{10}}{1-P_{00}} = 0{,}125$ in non-base airports.

The dependence of $P(\bar{A}_2)$ shown in Figure 3.10 reflects the influence of the justification level of the list of permissible failures (in this case, according to the parameter $\frac{P_{32}}{P_{30}}$, i.e., failures for which continuation of flight to the base airport is allowed, relative to failures) on the dynamics of $P(A_2)$ with respect to the parameter $\frac{V_B}{V}$, in the absence of permissible failures that allow continuation of flights to the nearest scheduled maintenance (i.e., $P_{10} = 0$). Since the nature of this dependence is not trivial, its interpretation will be carried out in somewhat more detail compared to the previous analyses.



$$P_{00} = 0{,}92; P_{10} = 0\ n - var; \frac{P_{20}}{P_{30}} = 0{,}3333;\ P(\bar{A}_2)\ at\ \frac{P_{32}}{P_{30}} = 1$$

3.10. fig. Dynamics of changes in the probability of state S_3 at various values of the parameter $\frac{P_{32}}{P_{30}}$ and with $P_{10} = 0$ in non-base airports.

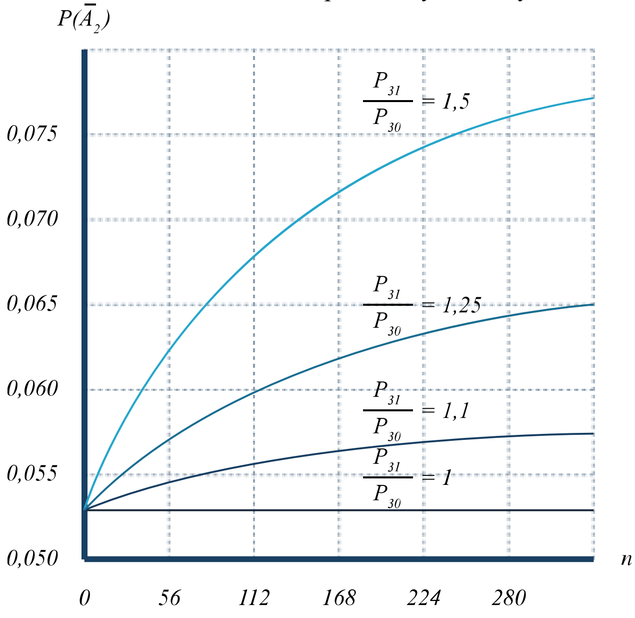
As shown in Figure 3.10, the values of $P(\bar{A}_2)$, in comparison to the case when $\frac{P_{32}}{P_{30}} = 1$, increase with the growth of the parameter $\frac{P_{32}}{P_{30}}$. Moreover, the rate of increase of $P(\bar{A}_2)$ becomes higher as the number of landings per flight in non-base airports increases (i.e., as the parameter $\frac{V_B}{V}$ decreases).

Compared to the case when $\frac{P_{32}}{P_{30}} = 1$, the absolute increase in $P(\bar{A}_2)$ is not significant only a few percentage points. This is due to the durations of the regeneration cycles of states S_2 and S_3.

To clarify the mechanism behind the observed change in $P(\bar{A}_2)$ compared to the case when $\frac{P_{32}}{P_{30}} = 1$, we refer to the graphs shown in Figures 2.3 and 2.5. The regeneration cycle of state S_2 corresponds to an entire flight leg, while that of state S_3 corresponds to a single flight. Therefore, the transition of an aircraft to state S_2 (determined by probability P_{20}) does not lead to a significant increase in within-leg transitions from S_2 to S_3 compared to the direct transitions from S_0 to S_3, even with substantial (1.5 times) increases in the parameter $\frac{P_{32}}{P_{30}}$.

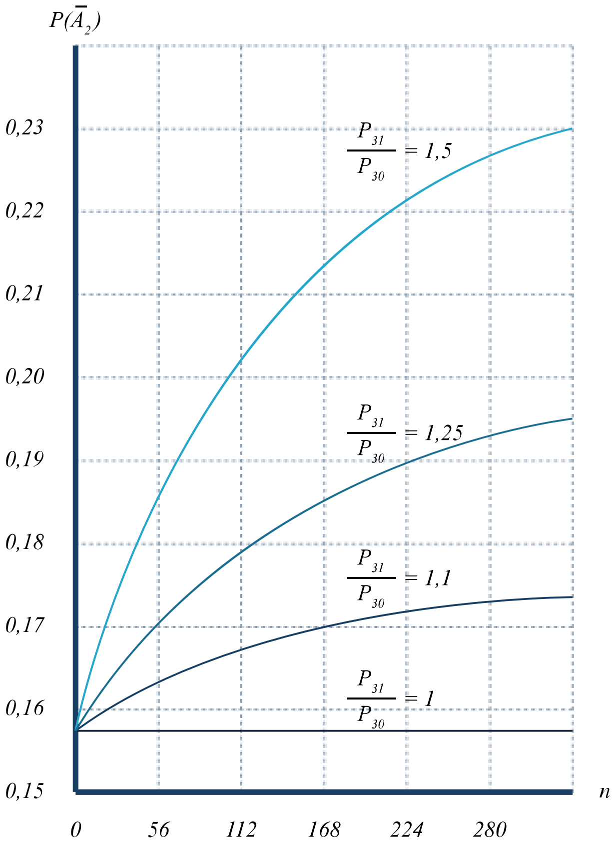
The dependencies for $P(\bar{A}_2)$, presented in Figures 3.11 to 3.13, reflect the influence of the justification level of the minimum equipment list (MEL) according to the parameter $\frac{P_{31}}{P_{30}}$ (i.e., failures that allow continued flight until the next scheduled maintenance check, relative to the failures typical for state S_3).

Unlike Figure 3.10, here we observe a dynamic behavior of $P(\bar{A}_2)$ with respect to the number of flights n. This is associated with the previously noted dynamics of state S_1.



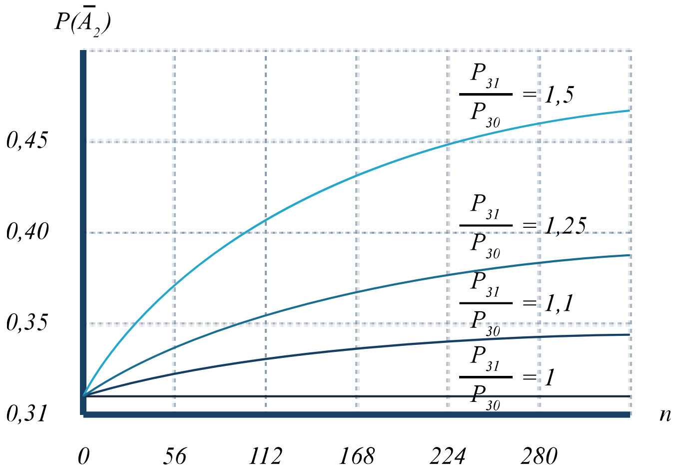
$$P_{00} = 0{,}92; \frac{P_{10}}{1 - P_{00}} = 0{,}125; \frac{P_{20}}{P_{30}} = 0{,}3333; \frac{P_{21}}{P_{20}} = 1; \frac{P_{32}}{P_{30}} = 1; \frac{V_B}{V} = 0{,}5$$

3.11. fig. Dynamics of changes in the probability of state S_3 at different values of the parameter $\frac{P_{31}}{P_{30}}$ in non-base airports for $\frac{V_B}{V} = 0{,}5$.



$$P_{00} = 0{,}92; \frac{P_{10}}{1 - P_{00}} = 0{,}125; \frac{P_{20}}{P_{30}} = 0{,}3333; \frac{P_{21}}{P_{20}} = 1; \frac{P_{32}}{P_{30}} = 1; \frac{V_B}{V} = 0{,}25$$

3.12. fig. Dynamics of changes in the probability of state S_3 at different values of the parameter $\frac{P_{31}}{P_{30}}$ in non-base airports for $\frac{V_B}{V} = 0{,}25$.



$$P_{00} = 0{,}92; \frac{P_{10}}{1 - P_{00}} = 0{,}125; \frac{P_{20}}{P_{30}} = 0{,}3333; \frac{P_{21}}{P_{20}} = 1; \frac{P_{32}}{P_{30}} = 1; \frac{V_B}{V} = 0{,}1429$$

3.13. fig. Dynamics of changes in the probability of state S_3 at different values of the parameter $\frac{P_{31}}{P_{30}}$ in non-base airports for $\frac{V_B}{V} = 0{,}1429$.

Compared to the case where $\frac{P_{31}}{P_{30}} = 1$, the absolute increase in $P(\bar{A}_2)$ is an order of magnitude higher than the increase observed in Figure 3.10. The influence of the number of landings per flight (parameter $\frac{V_B}{V}$) is manifested in the increase of the absolute values of $P(\bar{A}_2)$, and the nature of these changes is similar to those observed in Figures 3.9 and 3.10.

The identified difference in the nature of the influence of the justification levels of the lists of allowable failures those that permit continued flight until the next scheduled maintenance (parameter $\frac{P_{31}}{P_{30}}$) and those that permit continued flight to the base airport (parameter $\frac{P_{32}}{P_{30}}$) indicates a fundamental distinction in the approaches to the formation and justification of the lists of allowable failures for states S_1 and S_3.

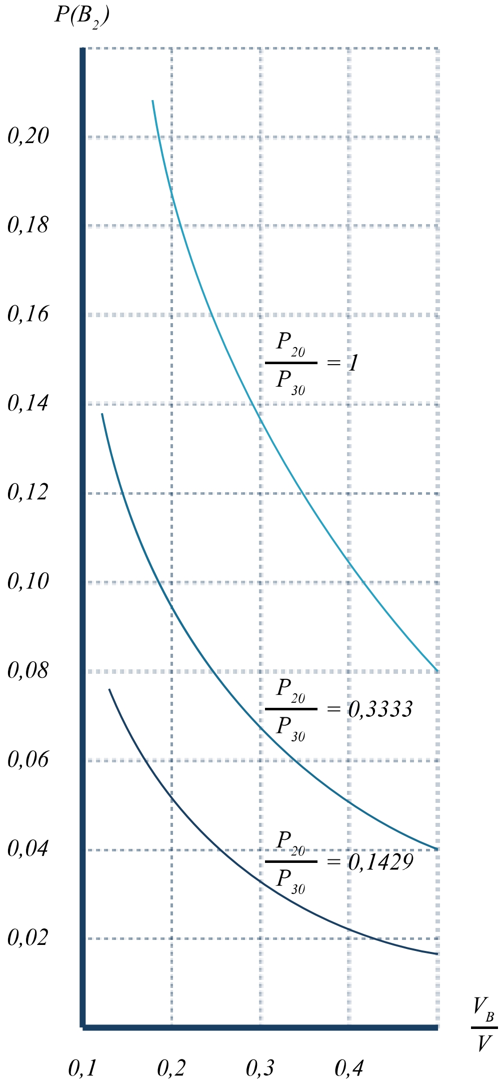
Let us now proceed to the analysis of the dynamics of the aircraft condition in the base airport, which is related to the results of the previously conducted analysis of the aircraft condition dynamics in outstation airports through the regeneration cycles of states S_1 and S_2.

As shown in Figure 3.14, the probability $P(B_2)$ of the aircraft arriving from a flight to the base airport in state S_2 varies only with the parameter $\frac{V_B}{V}$. An increase in the number of landings per flight at outstation airports (i.e., a decrease in the parameter $\frac{V_B}{V}$) leads to an increase in the probability $P(B_2)$. Moreover, an increase in the proportion of failures in the allowable list, with which continuation of flights to the base airport is permitted (an increase in the parameter $\frac{P_{20}}{P_{30}}$), all else being equal, leads to a steeper gradient of changes in the probability $P(B_2)$ with respect to the parameter $\frac{V_B}{V}$.

The introduction of allowable failures into the list failures with which continuation of flights to the nearest scheduled maintenance (increase in the parameter $\frac{P_{10}}{1-P_{00}}$) is permitted while maintaining the overall failure rate $(1 - P_{00})$ per flight on the aircraft, leads to a rather small decrease in the probability $P(B_2)$ (Figure 3.15), which increases with the growth of the parameter $\frac{V_B}{V}$. At the same time, both the gradient and the absolute magnitude of this decrease in the probability $P(B_2)$ increase as the parameter $\frac{P_{10}}{1-P_{00}}$ grows.

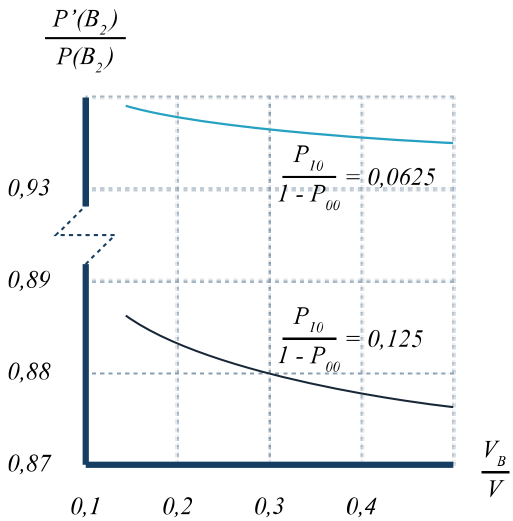
The dependence of the probability $P(B_1)$ of the aircraft arriving from a flight to the base airport in state S_1 on the parameter n (number of completed flights), shown in Figure 3.16, for various values of the parameters $\frac{V_B}{V}$ and $\frac{P_{10}}{1-P_{00}}$, and all other conditions being equal, demonstrates the following:

- The validity of the earlier conclusion drawn from Figures 3.2 and 3.3 about the increase in the probability of the aircraft having failures that allow continuation of flights until the next scheduled maintenance (periodic overhaul) as the number of completed flights increases (i.e., as the remaining resource until the next periodic maintenance decreases);
- The increase in the probability $P(B_1)$ with the growth of the parameter $\frac{P_{10}}{1-P_{00}}$, as well as the parameter $\frac{V_B}{V}$. At the same time, as seen in Figure 3.14, the influence of the parameter $\frac{P_{10}}{1-P_{00}}$ on the probability $P(B_1)$ is more significant compared to the influence of the parameter $\frac{V_B}{V}$.



$$P_{00} = 0{,}92; P_{10} = 0; \frac{P_{32}}{P_{30}} = 1; n - var$$

3.14. fig. Dynamics of changes in the probability of state S_2 at various values of the parameter $\frac{P_{20}}{P_{30}}$ with $P_{10} = 0$ at the base airport.



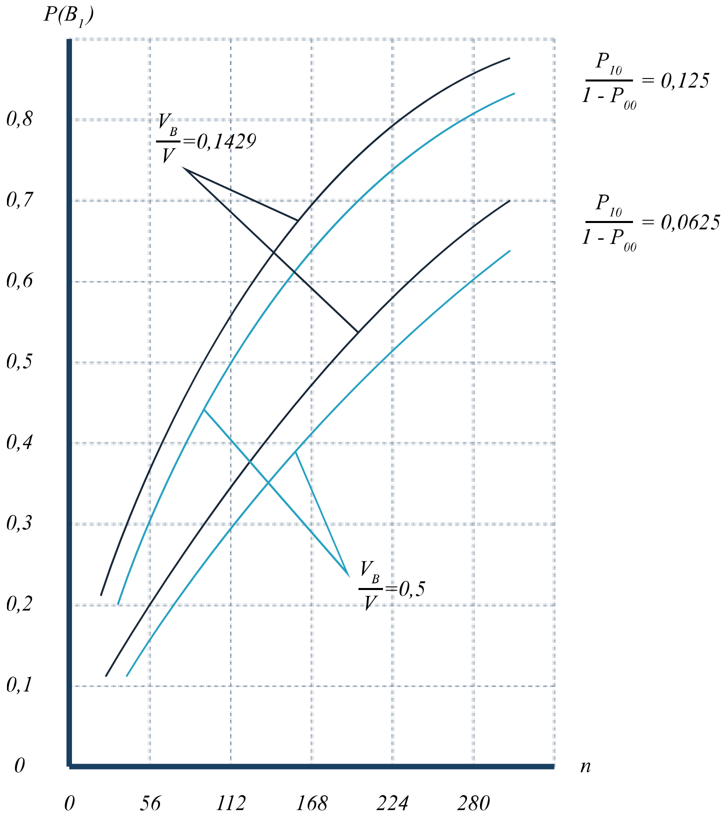
$P_{00} = 0{,}92; \frac{P_{20}}{P_{30}} = 0{,}1429; \frac{P_{21}}{P_{20}} = 1; \frac{P_{31}}{P_{30}} = 1; \frac{P_{32}}{P_{30}} = 1; n - var;\ P(B_2)$ at $P_{10} = 0$

3.15. fig. Dynamics of changes in the probability of state S_2 at various values of the parameter $\frac{P_{10}}{1-P_{00}}$ at the base airport.

Figure 3.17 shows, in the case where the list of allowable failures contains only failures that permit continuation of flights to the base airport, the influence of the justification level of this list relative to failures characteristic of state S_3 (parameter $\frac{P_{32}}{P_{30}}$). As seen in this figure, the probability $P(B_2)$ increases with the growth of the parameter $\frac{V_B}{V}$. Moreover, the gradient of this increase in probability $P(B_2)$ grows with an increase in parameter $\frac{P_{32}}{P_{30}}$. It should also be noted that the influence of parameter $\frac{P_{32}}{P_{30}}$ on the probability $P(B_2)$ (see Figure 3.17) is comparable to its influence on the probability $P(A_2)$ (see Figure 3.10).

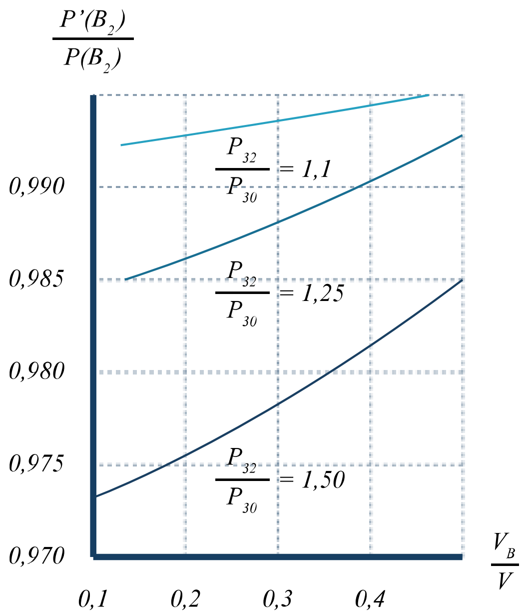
The probability $P(B_3)$ of the aircraft arriving from a flight to the base airport in state S_3, given a justified list of allowable failures with parameters $\frac{P_{31}}{P_{30}} = 1$ and $\frac{P_{32}}{P_{30}} = 1$, depends (see Figure 3.18) only on the distribution of the total failure flow $(1 - P_{00})$ per flight among states S_1, S_2, and S_3.

The dynamics of the probability $P(B_3)$ with respect to the parameter $\frac{V_B}{V}$, depending on the level of justification of the list of allowable failures that permit continued aircraft operation to the base airport (parameter $\frac{P_{32}}{P_{30}}$), are shown in Figure 3.19. For $\frac{P_{32}}{P_{30}} > 1$, there is a slight relative increase in the probability $P(B_3)$, the gradient of which grows with an increase in the parameter $\frac{P_{32}}{P_{30}}$.



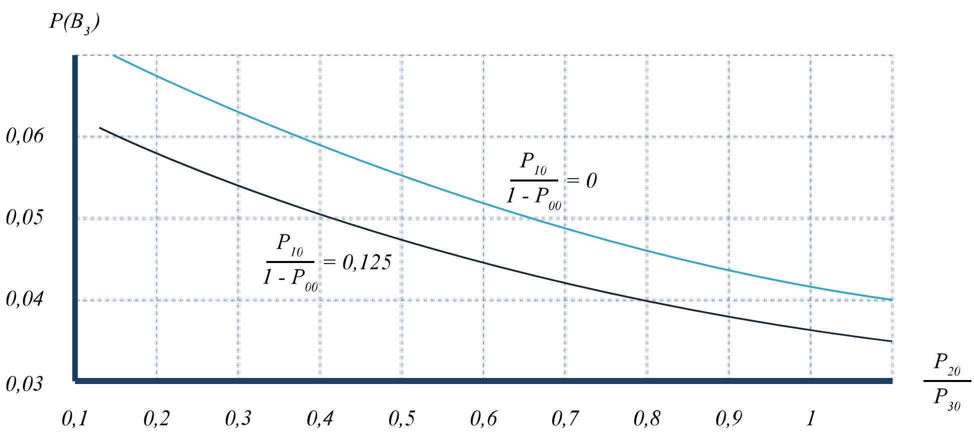
$$P_{00} = 0{,}92;\ \frac{P_{20}}{P_{30}} = 0{,}1429; \frac{P_{21}}{P_{20}} = 1; \frac{P_{31}}{P_{30}} = 1; \frac{P_{32}}{P_{30}} = 1$$

3.16. fig. Dynamics of changes in the probability of state S_1 at various values of the parameters $\frac{V_B}{V_n}$ and $\frac{P_{10}}{1-P_{00}}$ in the base airport.



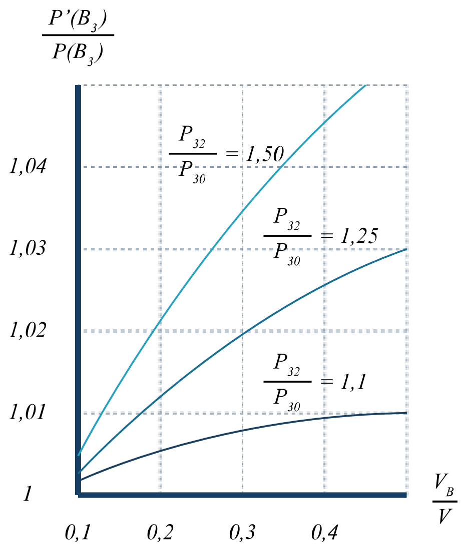
$$P_{00} = 0{,}92; P_{10} = 0; \frac{P_{20}}{P_{30}} = 0{,}3333; n - var; P(B_2) \text{ at } \frac{P_{32}}{P_{30}} = 1$$

3.17. fig. Dynamics of changes in the probability of state S_2 at various values of the parameter $\frac{P_{32}}{P_{30}}$ in the base airport.



$$P_{00} = 0{,}92; \frac{P_{31}}{P_{30}} = 1; \frac{P_{32}}{P_{30}} = 1; \frac{P_{21}}{P_{20}} - var; n - var; \frac{V_B}{V} - var$$

3.18. fig. Dynamics of changes in the probability of state S_3 at various values of the parameter $\frac{P_{10}}{1-P_{00}}$ in the base airport.



$P_{00} = 0{,}92; P_{10} = 0; \frac{P_{20}}{P_{30}} = 0{,}3333; P(B_3)$ at $\frac{P_{32}}{P_{30}} = 1$

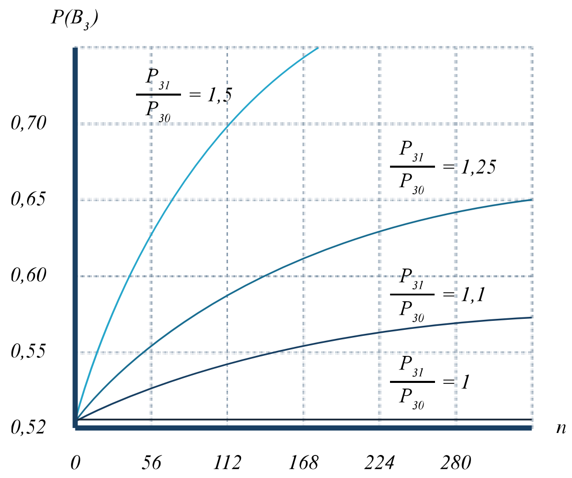
3.19. fig. Dynamics of changes in the probability of state S_3 at various values of the parameter $\frac{P_{32}}{P_{30}}$ in the base airport.

Figure 3.20 shows the dependence reflecting the dynamics of the probability $P(B_3)$ with respect to parameter n, depending on the level of justification of the list of allowable failures that permit continued operation of the aircraft until the next scheduled maintenance, relative to failures characteristic of state S_3 (parameter $\frac{P_{31}}{P_{30}}$).

The emergence of dynamics in the probability $P(B_3)$ over parameter n is associated with the state dynamics noted above. It should be noted that the influence of parameter $\frac{P_{31}}{P_{30}}$ on $P(B_3)$ is an order of magnitude greater than that of parameter $\frac{P_{32}}{P_{30}}$, and the mechanism of this influence coincides with the one observed earlier for the probability $P(\bar{A}_3)$.

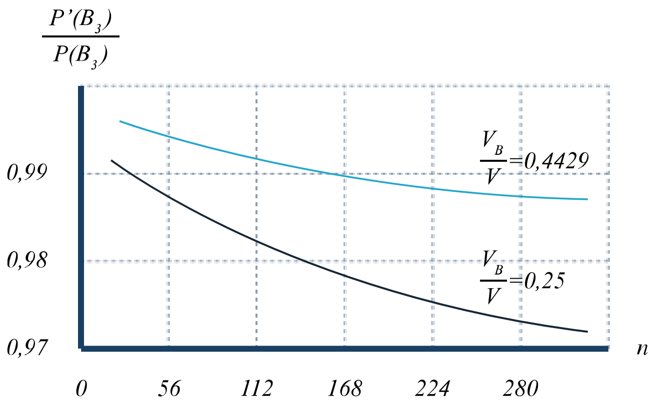
As shown in Figure 3.21, the effect of the parameter $\frac{P_{31}}{P_{30}}$ on the probability $P(B_3)$ practically does not depend on the number of landings per flight at non-base airports (parameter $\frac{V_B}{V}$). That is, the structure of the airline network on which the aircraft operates does not have a significant influence on the impact of parameter $\frac{P_{31}}{P_{30}}$ on the probability $P(B_3)$.

It should be noted that this nature of the influence of parameter $\frac{V_B}{V}$ on the changes of parameter $\frac{P_{31}}{P_{30}}$ regarding the probability $P(B_3)$ differs from what occurs for the probability $P(\bar{A}_2)$ (Figures 3.11–3.13).



$$P_{00} = 0{,}92; \frac{P_{10}}{1 - P_{00}} = 0{,}125; \frac{P_{20}}{P_{30}} = 0{,}3333; \frac{P_{21}}{P_{20}} = 1; \frac{P_{32}}{P_{30}} = 1; \frac{V_B}{V} = 0{,}5$$

3.20. fig. Dynamics of changes in the probability of state S_3 at various values of the parameter $\frac{P_{31}}{P_{30}}$ in the base airport.



$$P_{00} = 0{,}92; \frac{P_{10}}{1-P_{00}} = 0{,}125; \frac{P_{20}}{P_{30}} = 0{,}3333; \frac{P_{21}}{P_{20}} = 1; \frac{P_{32}}{P_{30}} = 1; \frac{P_{31}}{P_{30}} = 1{,}5; \ P(B_3) \text{ at } \frac{V_B}{V} = 0{,}5$$

3.21. fig. Dynamics of changes in the probability of state S_3 at various values of the parameter $\frac{V_B}{V}$ in the base airport.

Summarizing the above results of the analysis of the dynamics of aircraft states at base and non-base airports, the following important conclusions can be drawn from the perspective of ensuring the technical operation (TO) of aircraft:

- There are differences in the influence of the list of allowable failures (both up to the next scheduled maintenance and up to the base airport) on the number of situations that create a risk of aircraft departure delay at base and non-base airports. These differences are related to the regeneration cycles of states S_1, S_2 and S_3 during the technical operation process, as well as to the structure of the airline network on which the aircraft are operated.

- The concept of "justification of the list of allowable failures" is not simple and allows decomposition into at least three components (with respect to the parameters $\frac{P_{21}}{P_{20}}$, $\frac{P_{31}}{P_{30}}$ and $\frac{P_{32}}{P_{30}}$), each of which has a different impact on the dynamics of aircraft states during the maintenance process, and consequently on the situations that create the risk of aircraft departure delays at both base and non-base airports.
- The most significant impact on the dynamics of the aircraft state, which causes situations creating the risk of departure delays, is the level of justification of the list of allowable failures that permit continuation of flights up to the nearest scheduled maintenance (state S_1), relative to failures characteristic of state S_3.

Therefore, the introduction of such a list of allowable failures should primarily be justified from these positions. Otherwise, the use of such a list may not provide a measurable improvement under the given operational conditions.

The validity of MEL expansion was estimated based on failure transition ratios (P_{ij}) and reference thresholds derived from industry practice. Specifically, the probability of an S2-to-S3 transition during consecutive dispatches with unresolved MEL items was used as a risk indicator. Justification also relied on operational criteria from EASA Part-M/Part-CAMO and FAA AC 120-124 (CAMP), which define acceptable risk levels for deferred defects, emphasizing that such deferrals must not lead to a cumulative degradation of airworthiness or increased dispatch instability [28], [37].

3.2. Assessment of the significance of operational factors on the temporal indicators of flight regularity

The results of the analysis of the aircraft state dynamics presented in the previous section allow, using the formulas from section 2.2.4, to estimate the duration of aircraft departure delays for each type of airport in the airline network (T, TB, B). The dynamics of these indicators will be determined by the dynamics of states S_2 and S_3 for each airport type, as well as by the values of the parameters of these airports given in Table 1.4.

From a practical standpoint in managing the aircraft maintenance process on a given airline network, an important task is to assess the influence of each operational factor related both to the aircraft state dynamics (i.e., the matrix (2.4)) and the quantitative characteristics of the airline network structure on the duration of aircraft departure delays at both the base and non-base airports.

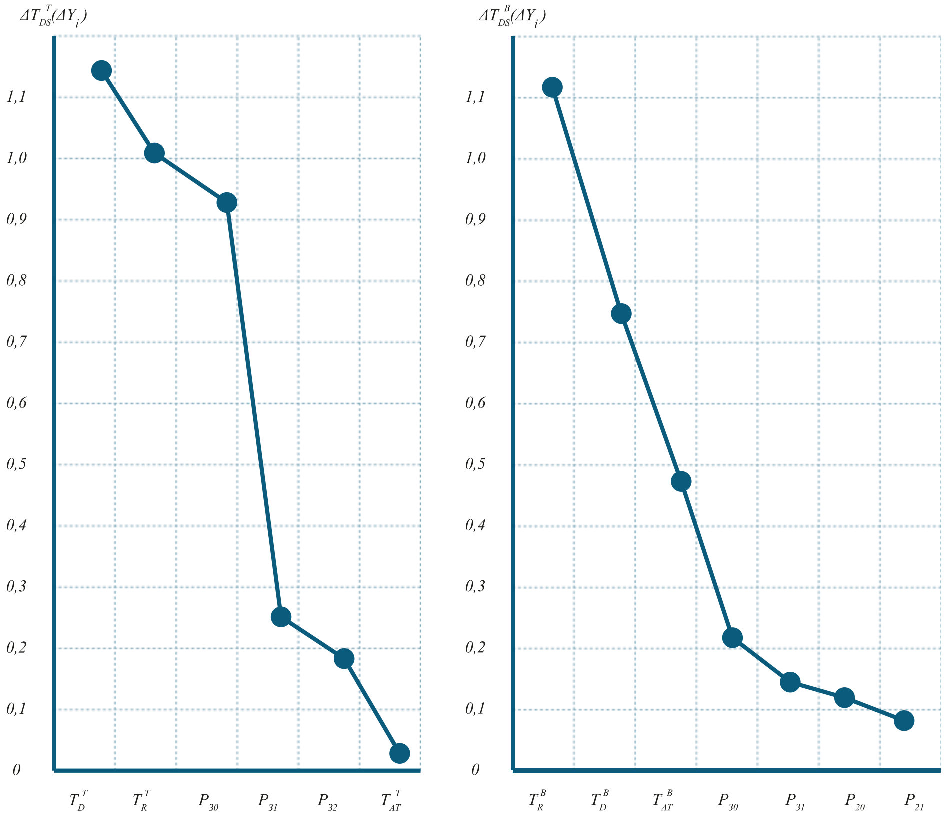
Solving this task will allow ranking these factors by their degree of influence, i.e., to prepare recommendations for the decision-maker regarding the appropriate sequence for implementing control actions.

It should be noted that, in addition to the above-mentioned time-based ranking of factors, the decision-maker also has cost (expense) assessments for each of them. Therefore, the cost-based ranking will, in general, differ from the ranking obtained by flight delay time. However, the essence of the ranking principle will remain unchanged. Thus, if a method for time-based ranking of factors is implemented, transitioning to cost-based ranking does not pose fundamental difficulties.

Let us formulate the problem of ranking factors. Let T_D be the total duration of aircraft departure delays at a given type of airports within a specified airline network over a given interval of operation time between periodic maintenance checks. Let Y be the set of operational factors influencing the value of T_D. The significance of the influence of an operational factor $Y_i \in Y$ on the value of T_D can be assessed by the following simple method:

- We select a sufficiently small, but significant with respect to changes in T_D, increment of the factor denoted as $\Delta Y = K_y \cdot Y$, where $0 < K_y < 1$;
- For each factor $Y_i \in Y$, we calculate the difference $\Delta T_D = T_D\left(Y_i^{\circ} + \Delta Y_i\right) - T_D(Y_i^{\circ})$ where Y_i° is the baseline value of the factor Y_i under specific operating conditions; $T_D(Y_i^{\circ})$ and $T_D\left(Y_i^{\circ} + \Delta Y_i\right)$ are the values of T_D corresponding to the factor Y_i taking values Y_i° and $Y_i^{\circ} + \Delta Y_i$ respectively.
- The ranking of factors $Y_i \in Y$ is arranged in descending order of the values of the difference ΔT_D.

Figure 3.22 presents, as an example, a polygon of factor significance distribution for a flight with one stop at an airport of type T, based on the given baseline values Y_i° of these factors. This figure provides a clear visual interpretation of the quantitative assessment results of the influence of each considered factor and does not require additional explanation.

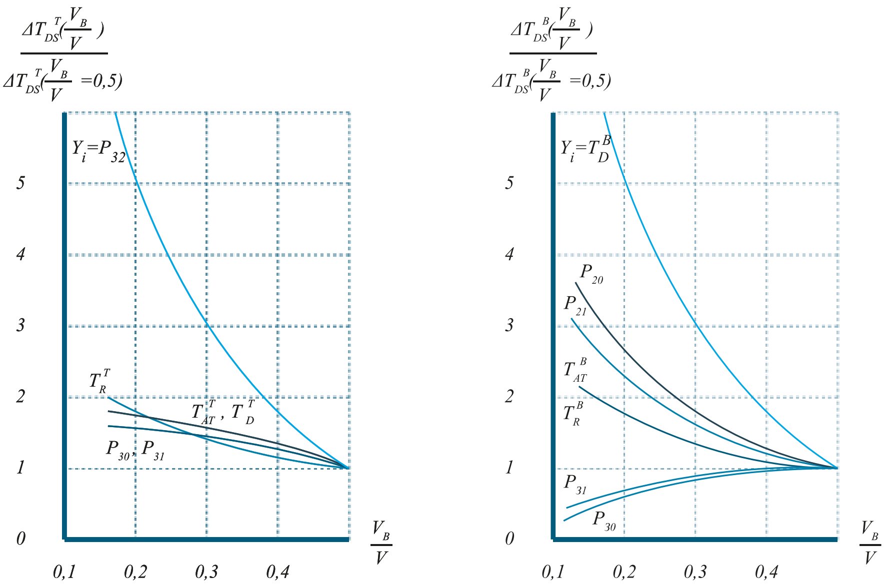


a) Airport of type T *b) Airport of type B*

$$P_{00} = 0{,}92;\ P_{10} = 0{,}01;\ P_{20} = 0{,}0175;\ P_{30} = 0{,}0525;\ P_{21} = 0{,}0175;\ P_{31} = 0{,}0525;\ P_{32} = 0{,}0525;\ T_{AT}^{T} = 0{,}5\ h.;\ T_{AT}^{B} = 0{,}5\ h.;\ T_{R}^{T} = 1{,}5\ h.;\ T_{R}^{B} = 1\ h.;\ T_{D}^{T} = 15\ h.;\ T_{D}^{B} = 15\ h.;\ P_{LB} = 0{,}5;\ P_{RES} = 0{,}5;\ \frac{P_{LB}}{P_{RES}} - const, n = 200;\ P_{SP}^{T} = 1; P_{SP}^{B} = 1$$

3.22. fig. Polygon of the distribution of significance of operational factors at the parameter value $\frac{V_B}{V} = 0{,}5$.

Figure 3.23 shows the dynamics of changes in the influence of the operational factors from Figure 3.22 depending on the parameter $\frac{V_B}{V}$. As can be seen from these figures, the dynamics of the factors' influence with respect to the $\frac{V_B}{V}$ parameter are not uniform. They are determined both by the previously described dynamics of aircraft states and by the baseline values Y_i° of the operational factors.



a) Airport of type T *b) Airport of type B*

3.23. fig. Dynamics of changes in the values of operational factors as a function of the parameter $\frac{V_B}{V}$ for the example shown in Figure 3.22.

Analysis of Figures 3.22 and 3.23 shows that the structure of the airline network has a significant impact on the ranking of operational factors affecting the duration of flight departure delays at airports within this network. Therefore, assessing the significance of each factor under the specific conditions of an airline's operations is not a trivial task and cannot be satisfactorily solved based solely on intuitive expert judgments. It requires extensive computational analysis.

In uncertain operational environments such as those involving technical delays, the ability to prioritize and make decisions under pressure becomes essential [45]. As a result, the proposed model, implemented as a specialized application tool, serves as a practical decision-support component within operational management.

3.3. Recommendations for improving the operational management of aircraft maintenance processes for the RAF-AVIA airline network

Development of recommendations for the RAF-AVIA airline is based on the ranking of operational factors that influence the duration of aircraft departure delays at airports within the airline's network.

The author collected and processed statistical data on the conditions of aircraft operation in RAF-AVIA, which are necessary for performing calculations using the developed model. These data include:

Transition probabilities P_{ij} between states per flight:

- P_{10} = 0,0325;
- P_{20} = 0,0325;
- P_{30} = 0,065;
- P_{21} = 0,033;
- P_{31} = 0,065;
- P_{32} = 0,065.

The structure of the route types in the network (Table 1.3) has the following probabilities (frequencies) of execution P_l over a calendar interval of one year (indices l correspond to the flight numbers in Table 1.3):

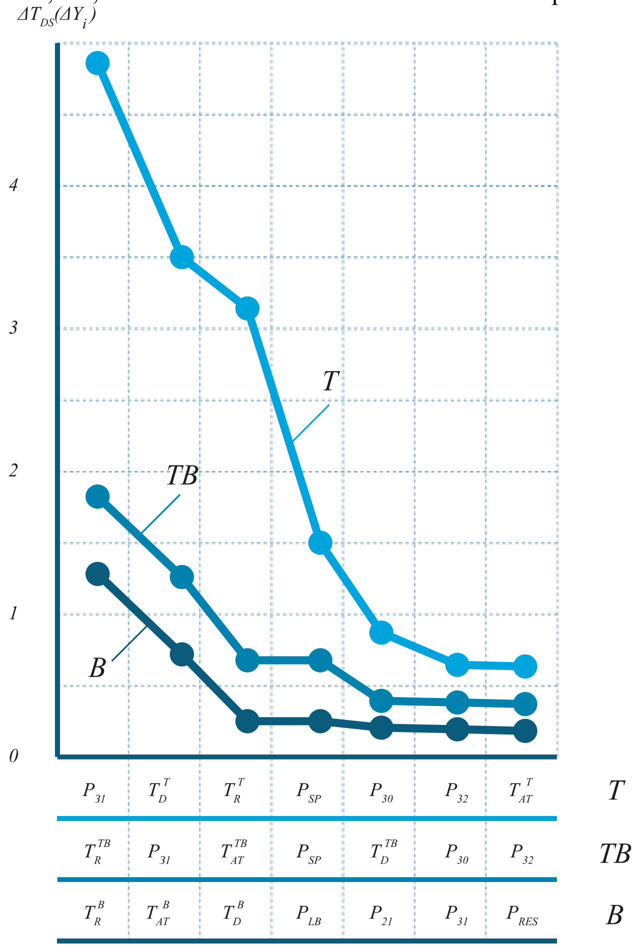
- P_1 = 0,062;
- P_2 = 0,141;
- P_3 = 0,071;
- P_4 = 0,085;
- P_5 = 0,076;
- P_6 = 0,139;
- P_7 = 0,178;
- P_8 = 0,101;
- P_9 = 0,102;
- P_{10} = 0,053;
- P_{11} = 0,062;
- P_{12} = 0,046.

Probabilistic characteristics of the aircraft recovery processes at the airline network airports (T, TB, B), as well as the processes of departure delay formation:

- Average ground time at airports of types T, TB, and B: $T_{AT}^{T} = 0{,}75\ h$; $T_{AT}^{TB} = 0{,}75\ h$; $T_{AT}^{B} = 1\ h$;
- Average aircraft recovery time at airports of types T, TB, and B: $T_{R}^{T} = 5\ h$; $T_{R}^{TB} = 3\ h$; $T_{R}^{B} = 1\ h$;
- Average time for express spare parts delivery at airports of types T, TB, and B: $T_{D}^{T} = 8\ h$; $T_{D}^{TB} = 1\ h$; $T_{D}^{B} = 0{,}5\ h$;
- Probability of availability of the spare parts required for aircraft recovery at airports of types T, TB, and B: $P_{SP}^{T} = 0{,}3\ h$; $P_{SP}^{TB} = 0{,}7\ h$; $P_{SP}^{B} = 0{,}95\ h$;
- Probability of aircraft rotation into the next flight at the base airport $P_{LB} = 0{,}3$.

Based on the average duration of a non-stop flight of an aircraft in the RAF-AVIA airline network, $T_{AN} = 2{,}09\ h$, and the average maintenance interval of approximately 500 hours, the value of the parameter R for conducting model-based calculations with the given network structure of RAF-AVIA is determined as $R = 51$.

In accordance with the approach described in Section 3.2, calculations were performed to assess the significance of the influence of operational factors on the duration of departure delays in airports of types T, TB, and B. The results of these calculations are presented in Figure 3.24.



$$P_{10} = 0{,}0325;\ P_{20} = 0{,}0325;\ P_{30} = 0{,}065;\ P_{21} = 0{,}033;\ P_{31} = 0{,}065;\ P_{32} = 0{,}065;\ T_{AT}^{T} = 0{,}75\ h.;\ T_{AT}^{TB} = 0{,}75\ h.;\ T_{AT}^{B} = 1\ h.;\ T_{R}^{T} = 5\ h.;\ T_{R}^{TB} = 3\ h.;\ T_{R}^{B} = 1\ h.;\ T_{D}^{T} = 8\ h.;\ T_{D}^{TB} = 1\ h.;\ T_{D}^{B} = 0{,}5\ h.;\ P_{SP}^{T} = 0{,}3;\ P_{SP}^{TB} = 0{,}7;\ P_{SP}^{B} = 0{,}95;\ P_{LB} = 0{,}3;\ P_{RES} = 0{,}6;\ n = 240$$

3.24. fig. Polygon of the distribution of the significance of operational factors for the RAF-AVIA airline network.

As shown in this figure, changes in operational factors significantly affect the duration of departure delays at Type T airports. Among all factors, those described by the parameters P_{31}, T_D^T and T_R^T have the greatest influence. The high significance of parameter P_{31} in Type T airports highlights the need for a thorough justification of any expansion of the list of allowable failures that permit continued operation until the next periodic maintenance, particularly in relation to failures characteristic of state S_3 (parameter $\frac{P_{31}}{P_{30}}$, whose baseline value in the calculations is 1). Potential management actions in the technical operation (TO) process for transit airports should focus on reducing the values of parameters T_D^T and T_R^T, as these are the most influential compared to others.

In transit-base airports (type TB), the most significant factors are those described by T_R^{TB} and P_{31}. The conclusion regarding the P_{31} parameter is similar to the one drawn for type T airports. Reducing aircraft recovery time T_R^{TB} in TB-type airports should be considered the most important measure for this type of airport.

For the base airport, the most significant factors are T_R^B and T_{AT}^B. Since the baseline values of these parameters used in the calculations were equal to 1 hour, reducing the aircraft recovery time T_R^B is more reasonable than increasing the ground time T_{AT}^B in the base airport during aircraft turnaround.

The recommendations described above were presented by the author to the management of RAF-AVIA. They were approved and adopted for implementation as part of the company's program for improving both technical operations and commercial performance.

4. OPTIMIZATION OF OPERATIONAL DECISIONS IN THE PRESENCE OF AOG-CRITICAL TECHNICAL DEFECTS

4.1. Renewal Theory in the Analysis of Aircraft Recovery Processes for Non-Dispatchable Defects

In this chapter, the problem of optimizing an airline's operational decision-making under the occurrence of technical defects leading to aircraft unavailability (AOG condition) is considered. Such defects are associated with an increased risk of departure delays or flight cancellations. The main focus is placed on the formalization of aircraft recovery processes and on assessing their impact on total technical delays under real route-based operational conditions. Based on the probabilistic models of aircraft technical state evolution and recovery processes developed in the previous chapters, an objective function is introduced, allowing a quantitative evaluation of the effectiveness of various operational and managerial decisions. The chapter demonstrates the application of the proposed approach using a practical airline route case study and analyzes the influence of operational, logistical, and organizational factors on the magnitude of technical delays, thereby providing a justification for the selection of optimal management strategies within the aircraft maintenance and operational control system.

Aircraft technical operation processes are characterized by recurring cycles of defect occurrence followed by restoration of airworthiness. In the case of defects classified as S_3 (Unserviceable), each such cycle results in a temporary withdrawal of the aircraft from operation and leads to operational losses. A formal description of such processes is provided within the framework of Renewal Theory.

Let us consider a sequence of random variables:

$$\{X_n\}_{n\geq 1}, \tag{4.1}$$

Where X_n denotes the duration of the n-th recovery cycle following the occurrence of an S_3 defect. The completion times of the recovery cycles are defined as follows:

$$S_n = \sum_{i=1}^{n} X_i \tag{4.2}$$

The recovery process is defined by the following expression:

$$N(t) = \max\{n \in N : S_n \leq t\} \tag{4.3}$$

This expression characterizes the number of completed recovery cycles over a time interval t and is referred to as the recovery process.

Where:

- X_n – duration of a single recovery cycle associated with an S_3 defect;
- S_n – completion time of the n-th recovery cycle;
- $N(t)$ – number of eliminated S_3 defects over the time interval t.

A fundamental characteristic of the renewal process is the renewal function:

$$m(t) = E[N(t)] \tag{4.4}$$

Within the framework of renewal theory, particular importance is attached to the asymptotic behavior of the average number of completed recovery cycles over a sufficiently long observation period. For a renewal process with independent and identically distributed cycle durations, provided that the mathematical expectation of a single cycle duration is finite, the following relationship holds:

$$\lim_{t\to\infty} \frac{m(t)}{t} = \frac{1}{E[X_1]} \tag{4.5}$$

Where:

- $E[X_1]$ – denotes the expected duration of a single recovery cycle associated with a defect of category S_3.

This relationship indicates that, over a long-time horizon, the expected number of completed recovery cycles grows linearly with time, while the rate of this growth is determined by the mean recovery time of a single defect. The result follows directly from the law of large numbers applied to the sum of recovery cycle durations S_n and is widely used in the analysis of operational systems with repetitive recovery structures [70].

For aircraft technical operation tasks, a particularly important extension of this model is the renewal reward process. Within this framework, each recovery cycle is associated with a random variable R_n, representing the operational losses caused by the occurrence of an S_3 category defect, primarily in the form of technical flight delays.

The cumulative operational losses over the time interval t are defined as:

$$Y(t) = \sum_{n=1}^{N(t)} R_n \tag{4.6}$$

Where:

- R_n – time losses (flight delays) caused by the n-th S_3 defect;
- $Y(t)$ – cumulative operational losses over the time interval t.

Provided that the mathematical expectations of the recovery cycle duration and the associated operational losses are finite, i.e. $\mathbb{E}[X_1] < \infty$ and $\mathbb{E}[R_1] < \infty$, the following relation holds for the renewal-reward process:

$$\lim_{t\to\infty} \frac{E[Y(t)]}{t} = \frac{E[R_1]}{E[X_1]} \tag{4.7}$$

This expression shows that the average operational losses per unit time are determined by the ratio of the expected loss caused by a single S_3 defect to the average recovery time of the aircraft. Thus, a direct relationship is established between the characteristics of the recovery process and the level of operational losses arising within the aircraft maintenance system [71].

Within the framework of the model developed in this study, the duration of a single recovery cycle is determined by the operational conditions and depends on the type of airport where the recovery activities are performed. For an airport of type $k \in \{T, TB, B\}$, the recovery cycle duration is defined as:

$$X^{(k)} = T_R^{(k)} + (1 - P_{SP}^{(k)}) \cdot T_D^{(k)} \tag{4.8}$$

Where:

- $T_R^{(k)}$ – average recovery time of an S_3 defect when the required spare parts are available;
- $P_{SP}^{(k)}$ – probability of spare parts availability at the maintenance location;
- $T_D^{(k)}$ – average delivery time of spare parts.

The losses R_n correspond to technical flight delays arising from the excess of the actual recovery time over the normative ground handling time.

Thus, defects of category S_3 are rigorously modeled as a renewal-reward process, which justifies the application of renewal theory for the assessment of operational losses.

4.2. Interpretation of Recovery Processes through Queueing Theory

Although renewal theory adequately describes individual cycles of S_3 defect rectification, under real operational conditions recovery activities are performed with limited resources. This leads to situations in which multiple S_3 defects compete for the same operational capabilities, resulting in waiting times for recovery.

To interpret this effect, the conceptual framework of Queueing Theory is employed.

Within queueing theory, a system is characterized by:

- an incoming flow of requests;
- a service process;
- the time a request spends within the system.

In the context of this study:

- a request corresponds to an aircraft affected by an S_3 defect;
- service represents the completion of recovery actions and the return of the aircraft to operation;
- waiting arises due to the unavailability of spare parts, repeated recovery cycles, and the absence of a reserve aircraft.

One of the fundamental relationships used in queueing theory is Little's Law:

$$L = \lambda W \tag{4.9}$$

Where:

- L – average number of aircraft undergoing recovery;
- λ – average rate of occurrence of S_3 defects;
- W – average time an aircraft remains in the S_3 state.

Little's Law follows from the principle of flow balance and is valid for stationary systems regardless of the specific distributions of interarrival and service times [72]. This property makes it suitable for the qualitative interpretation of systems with complex internal structures.

At the same time, classical queueing models such as *M/M/1* and *M/G/1* rely on a number of simplifying assumptions, including a Poisson arrival process, independent service times, and stationarity of system parameters. In this notation, the first letter *M* denotes a Markovian

(Poisson) arrival process, the second letter characterizes the statistical distribution of service times, and the digit *1* indicates the presence of a single service channel. In particular, the symbol G denotes a general (unspecified) probability distribution of service times, allowing for arbitrary statistical characteristics rather than assuming an exponential form. Despite this increased generality, the underlying assumptions of these models are not fully satisfied in the case of S_3 defects arising during aircraft operation [73].

In particular, the failure flow is governed by the matrix of probabilistic transitions P_{ij} between technical states rather than by an exogenous Poisson process. In addition, non-stationary effects related to aircraft ageing or seasonal operational patterns may be present, and recovery times are strongly influenced by logistical and organizational factors.

For these reasons, queueing theory is used in this study exclusively as an interpretative and supporting framework, rather than as an independent parametric model.

Within the developed model, the average time an aircraft remains in the S_3 state is expressed through the recovery parameters as:

$$W^{(k)} = T_R^{(k)} + (1 - P_{SP}^{(k)}) \cdot T_D^{(k)} \tag{4.10}$$

Thus, the impact of limited resources on delays is fully captured by the model parameters, without introducing any additional entities.

4.3. Objective Function for Optimizing the Recovery Process in the Presence of S_3 Defects

The objective of the optimization is to minimize the expected operational losses associated with defects of category S_3. To this end, an objective function based on the expected technical delay is introduced as an integral measure of operational efficiency.

For an airport of type $k \in \{T, TB, B\}$, the expected technical delay associated with a defect of category S_3 is defined as the difference between the expected total recovery time of the aircraft and the standard ground handling (turnaround) time:

$$\Delta T_{DS}^{(k)} = P_{SP}^{(k)} \cdot T_R^{(k)} + (1 - P_{SP}^{(k)}) \cdot (T_R^{(k)} + T_D^{(k)}) - T_{AT}^{(k)} \tag{4.11}$$

Where:

- $\Delta T_{DS}^{(k)}$ – denotes the expected technical delay caused by an S_3 defect at an airport of type k;
- $T_{AT}^{(k)}$ – is the standard turnaround time applicable to the given airport type.

If $\Delta T_{DS}^{(k)} < 0$, the technical delay is assumed to be zero. This corresponds to situations in which aircraft recovery is completed within the planned turnaround time and does not result in a departure delay.

Let a route r consist of a sequence of airports of different types k. The total expected technical delay for the route is then determined as the sum of the delays generated at each route node:

$$\Delta T_r = \sum_{k \in r} \Delta T_{DS}^{(k)} \tag{4.12}$$

Where:

- ΔT_r – is the total expected technical delay for route r.

To assess the operational efficiency of the airline's route network as a whole, an integral objective function is introduced. It is defined as the expected total technical delay over all considered route types:

$$F = \sum_{r=1}^{L} p_r \cdot \Delta T_r \tag{4.13}$$

Where:

- F – the optimization objective function;
- L – the number of route types considered;
- p_r – the probability of operating route r.

For base airports, the availability of a spare aircraft is additionally taken into account. The expected delay associated with the absence of a reserve aircraft is expressed as:

$$\Delta T_{RES} = (1 - P_{RES}) \cdot T_{RES} \tag{4.14}$$

Where:

- P_{RES} – the probability of spare aircraft availability;
- T_{RES} – the average waiting time in the absence of a reserve aircraft.

Furthermore, the model incorporates the probability of loop-back (return) scenarios, represented by the parameter P_{LB}, which reflects the repeated entry of an aircraft into the recovery cycle after completion of a previous repair.

Accordingly, the optimization problem is formulated as the minimization of the objective function over the set of controllable parameters:

$$\min_{\theta} F(\theta) \tag{4.15}$$

Where the vector of decision variables is defined as:

$$\theta = \{P_{ij}, T_R^{(k)}, T_D^{(k)}, T_{AT}^{(k)}, P_{SP}^{(k)}, P_{LB}, P_{RES}\} \tag{4.16}$$

Each managerial decision corresponds to a modification of one or more elements of the set θ and results in a quantitatively measurable change in the value of the objective function [74]. This enables a formal comparison of alternative operational strategies within the aircraft maintenance and operational management system.

4.4. Statistical Data on No-Dispatch Defects as the Basis for Applying the Optimization Model

The application of the model for assessing and optimizing operational losses caused by defects of category S_3, developed in this study, is not possible without a properly organized and methodologically justified statistical database. Unlike abstract analytical models intended primarily for theoretical investigations, the proposed approach is designed for practical use in an airline operational environment and therefore requires reliance on observed operational data.

In this context, statistical data do not serve as auxiliary illustrative material but represent an integral element of the model, providing:

- estimation of transition probabilities between aircraft technical states;
- determination of average recovery time characteristics;
- verification of the assumptions adopted in the theoretical part of the study;
- the possibility of quantitative comparison of management decisions through the value of the objective function.

Without systematic collection and analysis of statistical data, model parameters such as transition probabilities P_{ij}, average recovery times and logistics support times, as well as the probabilities of reserve and loop-back scenarios, cannot be reliably estimated or tested for stability. Consequently, the absence of a statistical basis makes practical application of the proposed method impossible and reduces the model to a purely theoretical construct.

A distinctive feature of aircraft operation is the event driven nature of defect occurrence, where each defect represents a separate operational event characterized by a specific time, location, and technical context. For this reason, within the framework of this study, statistical data are considered at the level of primary operational events recorded during aircraft operation rather than in the form of aggregated indicators. This approach makes it possible to preserve cause and effect relationships between the defect, the recovery process, and its operational consequences.

It is fundamentally important to note that defects of category S_3 do not form a homogeneous class in terms of their physical nature. Failures of generators, hydraulic valves, or avionics components belong to different technical subsystems and may exhibit different statistical properties. Therefore, when forming the statistical database, it is necessary to ensure the possibility of subsequent classification of defects within the S_3 category without compromising the integrity of the model or the correctness of data aggregation for optimization purposes.

Within this study, statistical data on S_3 defects are used to solve two interrelated tasks. First, they form the basis for estimating model parameters, including transition probabilities between technical states and time characteristics of recovery processes. Second, they allow assessment of the stability of statistical characteristics, which is a necessary condition for the legitimate use of mean values in the optimization objective function.

Thus, proper organization of data collection, storage, and processing related to S_3 defects is a mandatory prerequisite for applying the proposed method. The following subsections define the requirements for the structure of statistical data, the minimum sufficient sample size, and the principles of statistical data formation that ensure consistency with the theoretical model and practical applicability for optimizing operational processes.

The model for assessing and optimizing operational losses due to S_3 defects developed in this work belongs to the class of applied probabilistic-temporal models and is intended for use under real aircraft operating conditions. Unlike abstract analytical models, the parameters of this model are not defined a priori but are estimated based on actual operational data.

Within this study, operational statistics serve as the empirical foundation upon which all subsequent stages of analysis are built, including:

- formation of the transition probability matrix P_{ij};
- estimation of average recovery times $T_R^{(k)}$;
- estimation of average logistics support times $T_D^{(k)}$;
- determination of spare parts availability probabilities $P_{SP}^{(k)}$;

- estimation of loop-back probabilities P_{LB} and reserve aircraft usage probability P_{RES}.

Accordingly, without systematic accounting of S_3 defects, application of the model is not feasible, and any parameter values not supported by operational data cannot be considered justified.

In this work, statistical data are recorded at the level of individual operational events, where each event corresponds to the detection of a no-dispatch defect and the subsequent recovery process of the aircraft. This approach is adopted because:

- S_3 defects are discrete, event-driven phenomena;
- each defect is associated with a specific aircraft, airport, and time interval;
- the recovery process may follow different scenarios and result in different final technical states.

Event level statistical accounting allows preservation of causal relationships, correct identification of transitions between technical states, and elimination of distortions caused by premature data aggregation.

A unified primary event table formed during operation is used as the basic data collection instrument. Each row of the table corresponds to a single S_3 defect event.

The statistical table includes the following fields:

- Event identifier;
- Aircraft registration number;
- Manufacturer serial number (MSN);
- Date and time of defect detection;
- Airport of defect detection;
- Airport type (B, TB, T);
- Flight number and route segment;
- Defect category and ATA chapter;
- Brief defect description;
- Confirmation of transition to S_3 state;
- Scheduled turnaround time T_{AT};
- Spare parts availability at maintenance location;
- Start time of recovery actions;
- End time of recovery actions;
- Recovery duration T_R;
- Logistics support requirement;
- Spare part order time;
- Spare part receipt time;
- Logistics support duration T_D;
- Use of reserve aircraft;
- Loop-back event occurrence;
- Final technical state after recovery (S_0, S_1, S_2);
- Type of probabilistic transition (S_3 - S_j);
- Occurrence of flight delay;
- Actual technical delay duration;
- Calculated total recovery time T_{REC};
- Model-based delay value $\max(0, T_{REC} - T_{AT})$.

The transition probability matrix P_{ij} is formed directly from the statistical table by calculating relative frequencies of observed transitions:

$$P_{3j} = \frac{N_{3\to j}}{N_{S3}} \tag{4.17}$$

Where:

- $N_{3\to j}$ – is the number of events resulting in a transition from state S_3 to state S_j;
- N_{S3} – is the total number of registered S_3 defects.

Analogous procedures may be applied to transitions between other states if required for analysis.

Despite differences in the physical nature of defects (e.g., generator failure versus valve malfunction), aggregation of S_3 defects is justified within the proposed model because:

- all S_3 defects result in the same operational effect – no-dispatch condition;
- the optimization objective function depends on recovery and logistics times rather than on defect origin;
- differences between defect subclasses are reflected in distributions of T_R, T_D, and transition probabilities.

All parameters extracted from the statistical table are directly used in the computation of the optimization objective function, forming a closed-loop framework:

Operation → Statistics → Model Parameters → Objective Function → Management Decisions

Thus, statistical data on S_3 defects constitute not a supplementary element but a fundamental component of the proposed method, ensuring its practical applicability and methodological validity.

In this study, practical implementation of the developed model is demonstrated using statistical data related exclusively to failure events leading to the aircraft entering the S_3 (Aircraft on Ground, AOG) state. This choice is deliberate and methodologically justified, as the S_3 state represents the most critical operational condition directly affecting flight regularity and generating technical delays.

Statistical data on S_3 events used in this study are summarized in Table 4.1, which represents a unified log of technical events recorded during aircraft operation over the considered period. The table includes information on defect characteristics, detection conditions, recovery and logistics parameters, and actual operational consequences, enabling its use as an empirical basis for estimating model parameters and the objective function. Complete table presented in Appendix, Table A.1.

For the S_3 state, the influence of technical factors is explicitly manifested through exceeding the scheduled turnaround time by the total recovery time, which allows direct linkage between empirical data and the optimization objective function. In contrast, technical states S_1 and S_2 do not always result delays and are typically not associated with clearly identifiable operational consequences relevant to flight regularity indicators.

Transitions between states S_1 and S_2, as well as transitions from these states to S_0, are incorporated into the developed model in the form of probabilistic transition parameters obtained from aggregated operational and reliability statistics. However, detailed analysis of these data is not performed within the scope of the present practical example, as they do not contribute directly to technical delay magnitude and are not subject to optimization in this study.

Table 4.1

RAF-AVIA AOG Statistics Data Summary

Parameter	Value	Units/Notes
P_{SP}^{B}	0,950	Average logistics delivery time for missing spare parts to a base airport, minutes
T_{AT}^{B}	60	Average aircraft turnaround time at a base airport (excluding recovery actions), minutes
T_{R}^{B}	60	Average aircraft turnaround time at a base airport (excluding recovery actions), minutes
T_{D}^{B}	30	Average duration of aircraft technical recovery at a base airport when spares are available, minutes
P_{SP}^{TB}	0,700	Probability that required spare parts are immediately available on-site at a base airport
T_{AT}^{TB}	45	Average aircraft turnaround time at a transit-base airport (excluding recovery), minutes
T_{R}^{TB}	180	Average duration of aircraft technical recovery at a transit-base airport, minutes
T_{D}^{TB}	60	Average logistics delivery time for missing spare parts to a transit-base airport, minutes
P_{SP}^{T}	0,300	Probability that required spare parts are available at a transit airport
T_{AT}^{T}	45	Average aircraft turnaround time at a transit airport (excluding recovery), minutes
T_{R}^{T}	300	Average duration of aircraft technical recovery at a transit airport, minutes
T_{D}^{T}	480	Average logistics delivery time for missing spare parts to a transit airport, minutes
P_{LB}	0,30	Probability of aircraft loop-back into the next scheduled flight after an S3 event at the base airport
P_{RES}	0,60	Probability of aircraft loop-back into the next scheduled flight after an S3 event at the base airport
Outcome(S3→S0)	0,065	Conditional probability of restoring aircraft from state S3 to S0
Outcome(S3→S1)	0,065	Conditional probability of restoring aircraft from state S3 to S1
Outcome(S3→S2)	0,065	Conditional probability of restoring aircraft from state S3 to S2

Based on the data presented in Table 4.1, the following numerical values of model parameters were obtained for the analyzed operational period:

Scheduled turnaround time:
- T_{AT}^{T} = 45 min;
- T_{AT}^{TB} = 45 min;
- T_{AT}^{B} = 60 min.

Average recovery time:
- T_{R}^{T} = 300 min;
- T_{R}^{TB} = 180 min;
- T_{R}^{B} = 60 min.

Average logistics support time:
- T_{D}^{T} = 480 min;
- T_{D}^{TB} = 60 min;
- T_{D}^{B} = 30 min.

Spare parts availability probability:
- P_{SP}^{T} = 0,30;
- P_{SP}^{TB} = 0,70;
- P_{SP}^{B} = 0,95.

Loop-back probability:
- P_{LB} = 0,30.

Reserve aircraft availability probability:
- P_{RES} = 0,60.

Transition probabilities from state S_3:
- P_{30} = 0,065;
- P_{31} = 0,065;
- P_{32} = 0,065.

These values are used in calculating the total recovery time T_{REC}, the technical flight delay defined as:

$$\max(0, T_{REC} - T_{AT}) \tag{4.18}$$

and the optimization objective function.

4.5. Practical Application of the Developed Models and Calculation of the Objective Function for a RAF-AVIA Airline Route

In this subsection, a step-by-step example of applying the developed optimization approach to one of the RAF-AVIA airline routes is presented. The example integrates two components of the model:
- Model 1 – a model of changes in the aircraft technical state over a single flight (a Markov transition scheme between states S_0, S_1, S_2, S_3);
- Model 2 – a model of recovery processes and departure delay formation at airports of different types (parameters T_R, T_D, T_{AT}, P_{SP}).

The purpose of the example is to demonstrate, using a single route:
- how the probability distribution of aircraft technical states evolves along a sequence of flights according to Model 1;
- how these probabilities are linked to Model 2, which describes recovery processes and delay formation at airports of different types;
- how, on this basis, the objective function is formed and the expected total technical delay along the route is calculated.

All numerical values of the parameters (transition probabilities P_{ij}, recovery and delivery times, and probabilities of spare parts availability) are treated as statistical data of the airline.

For demonstration purposes, a route of type No. 7 (Table 1.3) is considered, consisting of a sequence of five flights:
- RIX (Riga, type B) → TLL (Tallinn, type TB);
- TLL (Tallinn, type TB) → WAW (Warsaw, type TB);
- WAW (Warsaw, type TB) → FRL (Forlì, type T);
- FRL (Forlì, type T) → SOF (Sofia, type TB);
- SOF (Sofia, type TB) → RIX (Riga, type B).

At each node of the route (TLL, WAW, FRL, SOF, RIX), the occurrence of recovery activities and the corresponding technical departure delay is possible.

Model 1: Step-by-step formation of the probability of entering state S_3

The technical condition of the aircraft (A/C) is described by four states:
- S_0 – serviceable condition;

- S_1 – permissible deviation (a condition allowed under the MEL (Minimum Equipment List – minimum equipment list));
- S_2 – more serious deviation requiring maintenance action;
- S_3 – AOG (Aircraft on Ground inability to dispatch the flight without recovery/rectification), associated with the risk of departure delay.

Based on the processing of the airline's operational statistics, the transition probabilities were estimated as follows:

- P_{10} = 0,0325;
- P_{20} = 0,0325;
- P_{30} = 0,065;
- P_{21} = 0,033;
- P_{31} = 0,065;
- P_{32} = 0,065.

The technical condition of the aircraft (A/C) is described by four states:

$$P = \begin{pmatrix} 0{,}87 & 0{,}0325 & 0{,}0325 & 0{,}065 \\ 0 & 0{,}902 & 0{,}033 & 0{,}065 \\ 0 & 0 & 0{,}935 & 0{,}065 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The element $P_{33} = 1$ is introduced not in the literal physical sense that "the aircraft remains AOG forever," but as a mathematical absorbing state that makes it possible to correctly accumulate the probability of reaching state S_3 over a sequence of flights. In other words, in this calculation S_3 is interpreted as an indicator state representing the occurrence of an AOG event within the considered flight chain. The actual recovery process is described by Model 2 and does not contradict the fact that, after rectification, the resulting technical state becomes S_0, S_1, or S_2.

The initial probability distribution prior to the first departure from the base airport RIX is assumed to be:

$$P^{(0)}(S_0) = 1, P^{(0)}(S_1) = 0, P^{(0)}(S_2) = 0, P^{(0)}(S_3) = 0$$

Thus:

$$P^{(0)} = (1,\ 0,\ 0,\ 0)$$

Step-by-step update of the probability distribution:

$$P^{(k)} = P^{(k-1)} \cdot P, k = 1, \dots, 5$$

Where:

- k – flight number in the sequence (1…5);
- $P^{(k)}$ – state probability distribution vector after completion of the *k*-th flight;
- P – transition probability matrix.

Let us denote:

$$P^{(k)} = (P^{(k)}(S_0), P^{(k)}(S_1), P^{(k)}(S_2), P^{(k)}(S_3))$$

and introduce the following key quantity:

$$q_k = P^{(k)}(S_3)$$

where q_k is the probability that, upon arrival at the corresponding airport in the route sequence, recovery is required (an AOG event occurs).

Flight 1: RIX (Riga, type B) → TLL (Tallinn, type TB)

$$P^{(1)} = (1{,}0{,}0{,}0) \cdot P$$

Components:
- $P^{(1)}(S_0) = 1 \cdot 0{,}87 = 0{,}87;$
- $P^{(1)}(S_1) = 1 \cdot 0{,}0325 = 0{,}0325;$
- $P^{(1)}(S_2) = 1 \cdot 0{,}0325 = 0{,}0325;$
- $P^{(1)}(S_3) = 1 \cdot 0{,}065 = 0{,}065.$

Thus:

$$P^{(1)} = (0{,}87,\ 0.0325,\ 0{,}0325,\ 0{,}065), q_1 = 0{,}065$$

Flight 2: TLL (Tallinn, type TB) → WAW (Warsaw, type TB)

$$P^{(2)} = P^{(1)} \cdot P$$

Components:
- $P^{(2)}(S_0) = 0{,}87 \cdot 0{,}87 = 0{,}7569;$
- $P^{(2)}(S_1) = 0{,}87 \cdot 0{,}0325 + 0{,}0325 \cdot 0{,}902 = 0{,}05759;$
- $P^{(2)}(S_2) = 0{,}87 \cdot 0{,}0325 + 0{,}0325 \cdot 0{,}033 + 0{,}0325 \cdot 0{,}935 = 0{,}059735;$
- $P^{(2)}(S_3) = 0{,}87 \cdot 0{,}065 + 0{,}0325 \cdot 0{,}065 + 0{,}0325 \cdot 0{,}065 + 0{,}065 \cdot 1 = 0{,}125775.$

Thus:

$$P^{(2)} = (0{,}7569,\ 0{,}05759,\ 0{,}059735,\ 0{,}125775), q_2 = 0{,}125775$$

Flight 3: WAW (Warsaw, type TB) → FRL (Forlì, type T)

$$P^{(3)} = P^{(2)} \cdot P$$

Components:
- $P^{(3)}(S_0) = 0{,}7569 \cdot 0{,}87 = 0{,}658503;$
- $P^{(3)}(S_1) = 0{,}7569 \cdot 0{,}0325 + 0{,}05759 \cdot 0{,}902 = 0{,}07654543;$
- $P^{(3)}(S_2) = 0{,}7569 \cdot 0{,}0325 + 0{,}05759 \cdot 0{,}033 + 0{,}059735 \cdot 0{,}935 = 0{,}08235195;$
- $P^{(3)}(S_3) = 0{,}7569 \cdot 0{,}065 + 0{,}05759 \cdot 0{,}065 + 0{,}059735 \cdot 0{,}065 + 0{,}125775 \cdot 1 = 0{,}18259963.$

Thus:

$$P^{(3)} = (0{,}658503,\ 0{,}07654543,\ 0{,}08235195,\ 0{,}18259963), q_3 = 0{,}18259963$$

Flight 4: FRL (Forlì, type T) → SOF (Sofia, type TB)

$$P^{(4)} = P^{(3)} \cdot P$$

Components:
- $P^{(4)}(S_0) = 0{,}658503 \cdot 0{,}87 = 0{,}57289761;$
- $P^{(4)}(S_1) = 0{,}658503 \cdot 0{,}0325 + 0{,}07654543 \cdot 0{,}902 = 0{,}02140134 + 0{,}069044 = 0{,}09044533;$
- $P^{(4)}(S_2) = 0{,}02140134 + 0{,}07654543 \cdot 0{,}033 + 0{,}08235195 \cdot 0{,}935 = 0{,}10092642;$
- $P^{(4)}(S_3) = 0{,}04280270 + 0{,}00497545 + 0{,}00535288 + 0{,}18259963 = 0{,}23573065.$

Thus:

$P^{(4)} = (0{,}57289761,\ 0{,}09044533,\ 0{,}10092642,\ 0{,}23573065), q_4 = 0{,}23573065.$

Flight 5: SOF (Sofia, type TB) → RIX (Riga, type B)

$$P^{(5)} = P^{(4)} \cdot P$$

Components:
- $P^{(5)}(S_0) = 0{,}57289761 \cdot 0{,}87 = 0{,}49842092;$
- $P^{(5)}(S_1) = 0{,}57289761 \cdot 0{,}0325 + 0{,}09044533 \cdot 0{,}902 = 0{,}10020086;$
- $P^{(5)}(S_2) = 0{,}57289761 \cdot 0{,}0325 + 0{,}09044533 \cdot 0{,}033 + 0{,}10092642 \cdot 0{,}935 = 0{,}11597007;$
- $P^{(5)}(S_3) = 0{,}57289761 \cdot 0{,}065 + 0{,}09044533 \cdot 0{,}065 + 0{,}10092642 \cdot 0{,}065 + 0{,}23573065 \cdot 1 = 0{,}28540816.$

Thus:

$P^{(5)} = (0{,}49842092,\ 0{,}10020086,\ 0{,}11597007,\ 0{,}28540816), q_5 = 0{,}28540816.$

Thus, according to Model 1, the following sequence of probabilities of occurrence of an AOG event S_3 at the route nodes is obtained:
- q_1 = 0,06500 (TLL, TB);
- q_2 = 0,12578 (WAW, TB);
- q_3 = 0,18260 (FRL, T);
- q_4 = 0,23573 (SOF, TB);
- q_5 = 0,28541 (RIX, B).

Model 2: Recovery Processes and Delay Formation at Airports Types B, TB, and T

Next, the expected departure delay is calculated for the case where recovery is required at an airport of a given type.

For each airport type, the following parameters are specified:
- T_{AT}^{loc} – standard turnaround time;
- T_{R}^{loc} – average recovery time when spare parts are available;
- T_{D}^{loc} – average spare parts logistics time (delivery);
- P_{SP}^{loc} – probability that the required spare parts are available at the maintenance location.

The following parameters are used (based on RAF-AVIA operational statistics):

- **RIX (B-Type):** $T_{AT}^{B} = 1{,}0$ h, $T_{R}^{B} = 1{,}0$ h, $T_{D}^{B} = 0{,}5$ h, $P_{SP}^{B} = 0{,}95$;
- **TLL, WAW, SOF (TB-Type):** $T_{AT}^{TB} = 0{,}75$ h, $T_{R}^{TB} = 3{,}0$ h, $T_{D}^{TB} = 1{,}0$ h, $P_{SP}^{TB} = 0{,}70$;
- **FRL (T-Type):** $T_{AT}^{T} = 0{,}75$ h, $T_{R}^{T} = 5{,}0$ h, $T_{D}^{T} = 8{,}0$ h, $P_{SP}^{T} = 0{,}30$.

The total recovery time for a specific event at an airport of type *loc* is defined as:

$$T_{REC}^{(loc)} = \begin{cases} T_{R}^{loc}, & if\ spare\ parts\ are\ available, \\ T_{R}^{loc} + T_{D}^{loc}, & if\ spare\ parts\ are\ not\ available. \end{cases} \tag{4.19}$$

The expected value of the total recovery time is defined as:

$$E[T_{REC}^{(loc)}] = P_{SP}^{loc} \cdot T_{R}^{loc} + (1 - P_{SP}^{loc}) \cdot (T_{R}^{loc} + T_{D}^{loc}). \tag{4.20}$$

Where:
- $E[\cdot]$ – expectation operator;
- $T_{REC}^{(loc)}$ – total recovery time;
- T_{R}^{loc} – recovery time;
- T_{D}^{loc} – delivery time;
- P_{SP}^{loc} – probability of spare parts availability;
- $loc \in \{B, TB, T\}$.

The expected departure delay at an airport of the given type:

$$E[\varphi^{(loc)}] = \max(0, E[T_{REC}^{(loc)}] - T_{AT}^{loc}), \tag{4.21}$$

Where:
- $\varphi^{(loc)}$ – technical departure delay;
- T_{AT}^{loc} – standard turnaround time.

Calculation of $E[\varphi^{(loc)}]$ for B/TB/T airport types

RIX (B-Type)

$$E[T_{REC}^{(B)}] = 0{,}95 \cdot 1{,}0 + 0{,}05 \cdot (1{,}0 + 0{,}5) = 1{,}025 \text{ h},$$
$$E[\varphi^{(B)}] = \max(0,\ 1{,}025 - 1{,}0) = 0{,}025 \text{ h} = 1{,}5 \text{ min}.$$

TLL, WAW, SOF (TB-Type)

$$E[T_{REC}^{(TB)}] = 0{,}7 \cdot 3{,}0 + 0{,}3 \cdot (3{,}0 + 1{,}0) = 3{,}3 \text{ h},$$
$$E[\varphi^{(TB)}] = \max(0,\ 3{,}3 - 0{,}75) = 2{,}55 \text{ h} = 153 \text{ min}.$$

FRL (T-Type)

$$E[T_{REC}^{(T)}] = 0{,}3 \cdot 5{,}0 + 0{,}7 \cdot (5{,}0 + 8{,}0) = 10{,}6 \text{ h},$$
$$E[\varphi^{(T)}] = \max(0,\ 10{,}6 - 0{,}75) = 9{,}85 \text{ h} = 591 \text{ min}.$$

4.6. Integration of the Models and Calculation of the Route Objective Function

At each route position, the expected technical delay is defined as the product of two components:

- q_k – the probability that recovery will be required at route position k, i.e. the occurrence of an AOG (Aircraft on Ground) event (the aircraft enters state S_3, meaning that the flight cannot be performed without corrective maintenance) at the moment of arrival at this route position (Model 1);
- $[\varphi^{(loc(k))}]$ – the expected technical delay associated with the recovery process at an airport of the corresponding type $loc(k) \in \{B, TB, T\}$ (Model 2).

$$E[\varphi_k] = q_k \cdot E\left[\varphi^{(loc(k))}\right] \tag{4.22}$$

Where:

- $E[\varphi_k]$ – the expected technical delay at route position k;
- q_k – the probability that the aircraft is in state S_3 at the moment of arrival at route position k;
- $loc(k)$ – the airport type associated with route position k.

Expected Delay by Airport Type (Based on Table 4.1)

Before proceeding to the analysis of a specific route, it is necessary to determine the baseline characteristics governing the formation of technical delays at airports of different types. Since the aircraft recovery process is strongly influenced by the infrastructural, logistical, and organizational capabilities of an airport, the model distinguishes three aggregated airport types: base (B), transit-base (TB), and transit (T) airports.

For each airport type, the following parameters are estimated on the basis of operational statistics (Table 4.1): the standard turnaround time, the average duration of recovery activities, the characteristics of spare parts logistics, and the probability of spare parts availability at the maintenance location. These parameters make it possible to determine the expected technical delay that arises when the aircraft transitions into state S_3 at an airport of the corresponding type.

At this stage, a local delay characteristic is formed, reflecting the consequences of a single AOG event at an airport of a given type, without accounting for the probability of its occurrence along the route. The resulting values are subsequently used as input parameters for calculating delays at individual route positions.

The total recovery time at an airport of type loc is represented as a random variable (using formulas 4.19-21):

Base airport (type B, RIX)

$$T_{AT}^{B} = 1{,}0\,h, T_{R}^{B} = 1{,}0\,h, T_{D}^{B} = 0{,}5\,h, P_{SP}^{B} = 0{,}95$$

$$E[T_{REC}^{(B)}] = 0{,}95 \cdot 1{,}0 + 0{,}05 \cdot (1{,}0 + 0{,}5) = 1{,}025\,h$$

$$E[\varphi^{(B)}] = \max(0, 1{,}025 - 1{,}0) = 0{,}025\,h = 1{,}5\,min$$

Transit–base airports (type TB: TLL, WAW, SOF)

$$T_{AT}^{TB} = 0{,}75\,h, T_R^{TB} = 3{,}0\,h, T_D^{TB} = 1{,}0\,h, P_{SP}^{TB} = 0{,}70$$

$$E[T_{REC}^{(TB)}] = 0{,}7 \cdot 3{,}0 + 0{,}3 \cdot (3{,}0 + 1{,}0) = 3{,}3\,h$$

$$E[\varphi^{(TB)}] = \max(0, 3{,}3 - 0{,}75) = 2{,}55\,h = 153\,min$$

Transit airport (type T, FRL)

$$T_{AT}^{T} = 0{,}75\,h, T_R^{T} = 5{,}0\,h, T_D^{T} = 8{,}0\,h, P_{SP}^{T} = 0{,}30$$

$$E[T_{REC}^{(T)}] = 0{,}3 \cdot 5{,}0 + 0{,}7 \cdot (5{,}0 + 8{,}0) = 10{,}6\,h$$

$$E[\varphi^{(T)}] = \max(0, 10{,}6 - 0{,}75) = 9{,}85\,h = 591\,min$$

Calculation of Delays at Route Positions (Model 1 + Model 2)

The next stage involves transitioning from generalized delay characteristics by airport type to the analysis of a specific airline route. Within the proposed approach, this is achieved by integrating two models: the probabilistic model of aircraft technical state evolution (Model 1) and the model describing recovery processes and delay formation at airports of different types (Model 2).

Model 1 makes it possible to determine the probability that, upon arrival at each **route position** along the route chain, the aircraft will be in state S_3, meaning that recovery will be required with an associated risk of departure delay. Model 2, in turn, defines the expected magnitude of the technical delay resulting from recovery at an airport of the corresponding type.

The integration of these two models allows the expected delay at each route position to be quantified by simultaneously accounting for both the probability of an AOG event and the severity of its operational consequences under specific recovery conditions. As a result, the calculation of delays at route positions represents a key linking stage between the probabilistic dynamics of aircraft technical states and the formation of the integrated route objective function.

Assignment of route positions:

- $k = 1$: TLL (TB);
- $k = 2$: WAW (TB);
- $k = 3$: FRL (T);
- $k = 4$: SOF (TB);
- $k = 5$: RIX (B).

Assignment of route positions:

- $q_1 = 0{,}06500$ (TLL, TB);
- $q_2 = 0{,}125775$ (WAW, TB);
- $q_3 = 0{,}182599625$ (FRL, T);
- $q_4 = 0{,}235730649375$ (SOF, TB);
- $q_5 = 0{,}285408157165625$ (RIX, B).

Using equation (4.19), the following results are obtained:

Route position 1 – TLL (TB):

$$E[\varphi_{TLL}] = 0{,}065 \cdot 2{,}55 = 0{,}16575\,h = 9{,}95\,min$$

Route position 2 – WAW (TB):

$$E[\varphi_{WAW}] = 0{,}125775 \cdot 2{,}55 = 0{,}32072625\, h = 19{,}24\, min$$

Route position 3 – FRL (T):

$$E[\varphi_{FRL}] = 0{,}182599625 \cdot 9{,}85 = 1{,}79860630625\, h = 107{,}92\, min$$

Route position 4 – SOF (TB):

$$E[\varphi_{SOF}] = 0{,}235730649375 \cdot 2{,}55 = 0{,}60111315590625\, h = 36{,}07\, min$$

Route position 5 – RIX (B):

$$E[\varphi_{RIX}] = 0{,}285408157165625 \cdot 0{,}025 = 0{,}00713520392914\, h = 0{,}43\, min$$

Route Objective Function (Baseline Scenario)

At the final stage of the baseline scenario analysis, an integrated performance indicator of route operation is formed the objective function, which represents the expected total technical delay over the entire route sequence. Unlike local delay estimates, the objective function makes it possible to consider the route as a single control object and to quantitatively assess the combined effect of technical failures and recovery processes.

The objective function is constructed as the sum of the expected delays at all route positions and reflects the contribution of each segment of the route to the overall disruption of flight regularity. This approach provides a clear physical interpretation of the result: the contribution of each route position is determined by the product of the probability of the aircraft entering state S_3 and the expected recovery-related delay at the corresponding airport type.

The resulting value of the objective function for the baseline scenario serves as a reference point for subsequent analysis and optimization. It is relative to this baseline value that the effectiveness of management decisions aimed at modifying recovery parameters, logistics, and the probabilistic dynamics of aircraft technical states is evaluated.

Objective function (expected total technical delay of the route):

$$J = E[\Phi^{route}] = \sum_{k=1}^{5} E[\,\varphi_k] = \sum_{k=1}^{5} q_k \cdot E[\varphi^{(loc(k))}] \qquad (4.23)$$

Substitution:

$$J_0 = 0{,}16575 + 0{,}32072625 + 1{,}79860630625 + 0{,}60111315590625 + 0{,}00713520392914 = 2{,}89333091608539\, h$$

In minutes:

$$J_0 \approx 2{,}89333091608539 \cdot 60 = 173{,}6\, min$$

4.7. Management Measures as Model Parameter Adjustments and Objective Function Minimization

In this subsection, the optimization problem is interpreted in terms of practical management measures that affect the parameters of the developed model. Each management action is formalized as a controlled modification of either the probabilistic component of the model, describing the likelihood of an aircraft entering the S_3 state, or the temporal component, characterizing recovery and logistical processes at airports of different types [75].

The objective of optimization is to minimize the expected total technical delay of the route, expressed by the objective function J, by selecting appropriate management strategies. The following analysis demonstrates how individual and combined measures influence the model parameters and lead to quantitative changes in the value of the objective function.

Optimization problem formulation:

$$\min_{u \in U} J(u) \tag{4.24}$$

where u – denotes a management strategy that modifies either the probabilistic component q_k (Model 1), the temporal component $E[\varphi^{(loc)}]$ (Model 2), or both simultaneously.

Below, four management measures are considered. For each measure, the corresponding changes in the model parameters are explicitly defined, after which the value of the objective function $J(u)$ is recalculated using the same analytical expressions as in the baseline scenario.

Measure 1. Increase in the Standard Turnaround Time at TB-Type Airports

This measure is based on a managerial decision to increase the planned (normative) ground handling time at transit–base (TB) airports. The rationale behind this approach is that a longer scheduled turnaround creates an operational buffer that partially absorbs recovery-related delays without affecting the departure time. Importantly, this measure does not influence the recovery process itself but modifies the reference level against which technical delays are assessed.

From the modeling perspective, the measure affects the parameter T_{AT}^{TB} while leaving the recovery and logistics parameters unchanged. As a result, the expected technical delay is reduced purely through scheduling flexibility rather than through improvements in maintenance or logistics performance.

Change: $T_{AT}^{TB}: 0{,}75 \to 1{,}0\ h$

Since the expected total recovery time at TB-type airports remains unchanged, $E[T_{REC}^{(TB)}] = 3{,}3\ h$, the expected technical departure delay for TB-type airports under Measure 1 is given by:

$$E[\varphi^{(TB)}]_{(1)} = \max(0, 3{,}3 - 1{,}0) = 2{,}3\ h$$

The cumulative probability of an AOG event occurring at TB-type airports along the route is obtained as the sum of the corresponding probabilities at the relevant route positions:

$$q_{TB} = q_1 + q_2 + q_4 = 0{,}065 + 0{,}125775 + 0{,}235730649375 = 0{,}426505649375$$

Thus:

$$J_1 = q_{TB} \cdot 2{,}3 + q_3 \cdot 9{,}85 + q_5 \cdot 0{,}025 \approx 2{,}7866\, h \approx 167{,}2\, min$$

Measure 2. Local Technical Support / AOG Stock at TB-Type Airports

This measure represents the establishment of enhanced local technical capabilities at TB-type airports, including the placement of critical spare parts (AOG stock) and improved availability of qualified maintenance personnel. Such organizational actions increase the probability that required spare parts are immediately available and reduce the average duration of recovery activities.

In the model, this measure simultaneously increases the spare-part availability probability P_{SP}^{TB} and decreases the mean recovery time T_R^{TB}. Consequently, it directly affects the temporal component of the objective function by reducing the expected delay associated with recovery events at TB airports.

Change: $P_{SP}^{TB}: 0{,}70 \rightarrow 0{,}90$, $T_R^{TB}: 3{,}0 \rightarrow 2{,}5\, h$ (for $T_D^{TB} = 1{,}0\, h$)

$$E[T_{REC}^{(TB)}]_{(2)} = 0{,}9 \cdot 2{,}5 + 0{,}1 \cdot (2{,}5 + 1{,}0) = 2{,}6\, h$$

$$E[\varphi^{(TB)}]_{(2)} = \max(0, 2{,}6 - 0{,}75) = 1{,}85\, h$$

Thus:

$$J_2 = q_{TB} \cdot 1{,}85 + q_3 \cdot 9{,}85 + q_5 \cdot 0{,}025 \approx 2{,}5947\, h \approx 155{,}7\, min$$

Measure 3. Engineer on Board

This measure reflects the operational practice of assigning a maintenance engineer to the flight crew, enabling in-flight troubleshooting, deferral of certain defects, and early corrective actions immediately after landing. As a result, the likelihood that a defect escalates to a non-dispatchable condition (state S_3, AOG) is reduced.

Within the modeling framework, this measure affects the probabilistic dynamics of technical state transitions (Model 1) by decreasing the probabilities q_k of entering state S_3 along the route. Unlike other measures, it does not alter recovery or logistics parameters but reduces the frequency of recovery events, leading to a global reduction of the objective function.

This measure affects Model 1: the probability of the aircraft reaching state S_3 at each route position is reduced across the entire route.

For the purposes of the calculation, a conservative reduction factor is assumed:

$$q_k^{(3)} = 0{,}6\, q_k (\text{i.e., a } 40\% \text{ reduction})$$

Since the objective function is linear in q_k, it follows that:

$$J_3 = \sum_{k=1}^{5} (0{,}6\ q_k) \cdot E[\varphi^{(loc(k))}] = 0{,}6\, J_0$$

$$J_3 = 0{,}6 \cdot 2{,}89333091608539 = 1{,}7360\, h \approx 104{,}2\, min$$

Measure 4. Express Spare-Part Delivery to a T-Type Airport

This measure targets the most critical logistical bottleneck identified in the route analysis: transit (T-type) airports with limited local support and long spare-part delivery times. By implementing express logistics solutions such as priority shipping, dedicated courier services, or pre-arranged supplier contracts the average delivery time of spare parts can be significantly reduced.

In the model, this measure decreases the logistics time parameter T_D^T for the T-type airport FRL, directly lowering the expected recovery time and, consequently, the technical delay associated with AOG events at this location.

Change: $T_D^T: 8{,}0 \to 2{,}0\, h$ (for $T_R^T = 5{,}0\, h, P_{SP}^T = 0{,}30$)

$$E[T_{REC}^{(T)}]_{(4)} = 0{,}3 \cdot 5{,}0 + 0{,}7 \cdot (5{,}0 + 2{,}0) = 6{,}4\, h$$

$$E[\varphi^{(T)}]_{(4)} = \max(0, 6{,}4 - 0{,}75) = 5{,}65\, h$$

Thus:

$$J_4 = q_{TB} \cdot 2{,}55 + q_3 \cdot 5{,}65 + q_5 \cdot 0{,}025 \approx 2{,}1264\, h \approx 127{,}6\, min$$

Selection of the optimal measure (single-strategy scenario)

A comparison of the objective function values (in minutes) gives:

- Baseline: $J_0 \approx 173{,}6$ min;
- Measure 1: $J_1 \approx 167{,}2$ min;
- Measure 2: $J_2 \approx 155{,}7$ min;
- Measure 3: $J_3 \approx 104{,}2$ min (best result among the single measures);
- Measure 4: $J_4 \approx 127{,}6$ min.

Therefore, within the class of single operational measures, the optimal choice is Measure 3 (engineer on board), as it yields the minimum value of the objective function $J(u)$.

Combining measures and the optimal combined strategy

Combining measures is necessary (and scientifically justified) because the measures act on different components of the objective function:

- Measure 3 reduces the probabilistic component q_k (Model 1);
- Measures 2 and 4 reduce the time component $E[\varphi^{(loc)}]$ (Model 2) for different airport types (TB and T, respectively).

Since:

$$J = \sum q_k \cdot E[\varphi^{(loc(k))}]$$

a simultaneous reduction of both q_k and $E[\varphi]$ produces a multiplicative effect.

Combination A: Measure 3 + Measure 4

This implies: $q_k^{(A)} = 0{,}6 q_k$ and $E[\varphi^{(T)}] = 5{,}65\, h$.

Because changing q_k scales the entire value of J_4, we obtain:

$$J_A = 0{,}6 J_4 = 0{,}6 \cdot 2{,}1264 = 1{,}2758\, h \approx 76{,}6\, min$$

Combination B: Measure 2 + Measure 4

Here q_k remains unchanged, but the expected delays are reduced for both TB and T airports:

$$J_B = q_{TB} \cdot 1{,}85 + q_3 \cdot 5{,}65 + q_5 \cdot 0{,}025 \approx 1{,}8279\, h \approx 109{,}7\, min$$

Combination C (most effective): Measure 3 + Measure 2 + Measure 4

That is: $q_k^{(C)} = 0{,}6 q_k$, $E[\varphi^{(TB)}] = 1{,}85\, h$, $E[\varphi^{(T)}] = 5{,}65\, h$.

First, we take the result of Combination B and then apply the scaling by q_k:

$$J_C = 0{,}6\, J_B = 0{,}6 \cdot 1{,}8279 = 1{,}0967\, h \approx 65{,}8\, min$$

Thus, the implementation of the proposed combination of operational management measures reduces the expected technical delay from approximately 173,6 minutes to 65,8 minutes for the considered route. This corresponds to a reduction of about 62% compared to the baseline operational scenario.

The obtained result demonstrates the cumulative effect of simultaneously reducing both components of the objective function. While Measure 2 and Measure 4 decrease the expected recovery-related delay by improving maintenance and logistics conditions, Measure 3 further amplifies this effect by reducing the probability of AOG occurrence along the entire route. As a result, the combined strategy leads to a substantially lower value of the objective function compared to any single or partially combined measure, confirming the efficiency of integrated operational decision-making.

At the same time, the obtained results indicate that the improvement in flight regularity should not be interpreted as a single fixed percentage value. Since the developed model evaluates the expected technical delay under different operational conditions, the achievable improvement depends on the selected management strategy and operational parameters. In the considered case study, individual management measures lead to different levels of delay reduction, while their combined implementation produces the most significant effect. In particular, the reduction of the expected technical delay varies from approximately 3.7% for the least influential measure (Measure 1) to about 62% for the optimal combined strategy. Thus, the achievable improvement in flight regularity should be interpreted as a range of possible results depending on the applied operational measures and management decisions.

4.8. Final conclusions based on the case study

This subsection presents the final conclusions drawn from the route-based case study and consolidates the results of the optimization analysis. The example demonstrates how the developed objective function links probabilistic aircraft state dynamics with recovery and logistics parameters, enabling a quantitative comparison of alternative management strategies. The findings confirm the validity of the proposed approach for evaluating and optimizing operational decisions in the presence of AOG-critical technical defects.

1. In the baseline scenario, the expected total technical delay of the route amounts to:

$$J_0 \approx 2{,}8933\, h \approx 173{,}6\, min$$

2. Among the single management measures considered, the minimum value of the objective function is achieved by Measure 3 (engineer on board):

$$J_3 \approx 1{,}7360\ h \approx 104{,}2\ min$$

3. At the same time, a scientifically justified combination of measures – simultaneously influencing both the probabilistic component q_k and the temporal component $E[\varphi^{(loc)}]$ – makes it possible to obtain a substantially better result. The most effective of the combined strategies considered is:

$$u^* = \{Measure\ 3 + Measure\ 2 + Measure\ 4\}$$

for which:

$$J(u^*) = J_C \approx 1{,}0967\ h \approx 65{,}8\ min$$

Therefore, within the framework of this example, the optimal management strategy is a combined strategy (Measure 3 + Measure 2 + Measure 4), as it ensures the minimum value of the objective function and, consequently, the greatest reduction in the expected technical delays along the route.

The step-by-step calculations performed demonstrate that the objective function developed in this study makes it possible to formally and quantitatively assess the impact of various management decisions on the total technical delay of a route. The baseline operating scenario for the RIX-TLL-WAW-FRL-SOF-RIX route is characterized by an expected delay of approximately 173,6 minutes, reflecting the combined influence of the probabilities of S_3 category defects and the limited recovery capabilities at TB and T type airports.

The analysis of individual management measures shows that the greatest single-measure effect is achieved by reducing the probability of an aircraft entering the S_3 state, which is implemented through operating the flight with an engineer on board. However, the maximum reduction of the objective function is achieved not by implementing a single measure, but by combining management actions that affect both the probabilistic and temporal components of the model.

The combined strategy, which includes:

- a reduction in the probabilities q_k (engineer on board);
- improved recovery conditions at TB-type airports;
- reduced logistics time at the critical T-type airport.

It makes possible to reduce the expected total technical delay of the route to approximately 65,8 minutes, i.e. by about 62% compared to the baseline scenario.

Thus, the approach developed in Chapter 4 provides not only a formal definition of an optimal management strategy, but also clearly demonstrates that optimization of operational processes in aviation should be treated as a problem of integrated impact on several interrelated factors, rather than as the selection of a single isolated measure. This confirms the practical applicability of the proposed model for decision support in the aircraft technical operation system.

RESEARCH RESULTS

1. A comprehensive analysis of the current state of the flight regularity problem in civil aviation and its impact on airline network operations was performed.
2. Aircraft failures and technical malfunctions during operation were analyzed, recovery capability within the airline network was assessed, and operational factors influencing technical delay probability were determined.
3. Mathematical models of aircraft state assessment and recovery processes in accordance with MEL (Minimum Equipment List) requirements within airline network airports were developed, aimed at reducing both the probability and duration of departure delays.
4. The FlightSync application tool was developed for implementing the proposed probabilistic mathematical models within operational aircraft maintenance management.
5. The FlightSync application tool was tested and validated using operational and statistical data from the RAF-AVIA airline.

CONCLUSION

This dissertation addresses the problem of improving flight regularity through the optimization of operational aircraft maintenance management within an airline route network.

The main results of the research are as follows:

1. A model of aircraft technical state evolution was developed based on discrete operational states and probabilistic transitions between them. The model accounts for in-flight degradation, ground-based recovery processes, and operational limitations related to MEL application.
2. A model of technical delay formation within the airline route network was constructed. The model integrates:
   - the probability of defect occurrence;
   - recovery time;
   - spare part availability;
   - airport type and its technical capability.
3. A parametric analysis was performed to assess the impact of key operational factors on expected technical delay. The most influential parameters affecting flight regularity were identified.
4. An objective function was developed to quantitatively evaluate management decisions under AOG-critical defect conditions. This made it possible to formally compare alternative operational strategies rather than relying on qualitative judgment.
5. The developed optimization approach was applied to a real airline route configuration of RAF-AVIA. The baseline expected total technical delay was quantified, and alternative management measures were formally evaluated. It was established that the greatest effect is achieved not by isolated actions, but by a coordinated combination of measures affecting both probabilistic and time-related parameters of the system, resulting in a reduction of expected technical delay by more than 60% compared to the baseline scenario.
6. The developed models were implemented in the FlightSync application tool, designed to calculate cumulative technical delays and analyze alternative management scenarios. The application tool was tested using RAF-AVIA operational data and can be used as a decision-support tool for airline engineering and operations control departments.

The results of this research demonstrate that improving flight regularity should be treated as a problem of integrated management of interrelated operational parameters. The proposed approach provides a quantitative basis for selecting optimal strategies in the presence of technical defects and is applicable to real airline operations.

LIST OF LITERATURE

1. EUROCONTROL. Airport CDM Implementation Manual. Brussels: EUROCONTROL, 2022. Pages: 15-26; 68-74.
2. McKinsey & Co. Airline Operations: Achieving On-Time Performance. 2019. Pages: 5-12; 29-37.
3. IATA. Delay Codes - Standard IATA Delay Codes. 2023 Edition. Geneva: IATA. Pages: 3-8; 12-16.
4. Castro A. J. M., Rocha A. P., Oliveira E. A New Approach for Disruption Management. Springer, 2015. Pages: 70-81. DOI: 10.1007/978-3-319-18944-4_6
5. SITA. Horizon Passenger Services System. Société Internationale de Télécommunications Aéronautiques, 2021.
6. EUROCONTROL. Guidelines for Air Traffic Flow and Capacity Management. Brussels: EUROCONTROL, 2020. Pages: 12-17; 46-58.
7. Skybrary. Delay Management in ATM. Brussels: SKYbrary Foundation, 2022. Pages: 4-9; 21-27.
8. Nowakowski J. Aircraft Reliability Analysis. Springer, 2021. Pages: 71-87; 134-142. ISBN: 9783030731190
9. Modarres M. Risk Analysis in Engineering. CRC Press, 2006. Pages: 144-157; 204-215. ISBN: 9780849322203
10. Morrell P. S. Airline Finance. Routledge, 2019. Pages: 116-124; 212-215. ISBN: 9781138365284
11. ICAO. Safety Report. 2023 Edition. Montreal: ICAO. Pages: 8-14; 28-35.
12. ATA. ATA Spec 100 & iSpec 2200. ATA, 2020. Pages: 12-16; 32-45.
13. ICAO. Annex 6 - Operation of Aircraft, Part I. 12th Edition. Montreal: ICAO, 2022. Pages: 58-66; 96-101
14. EUROCONTROL. Performance Review Report 2022. Brussels: EUROCONTROL, 2023. Pages: 10-15; 52-64.
15. Swissinfo. Punctuality: SWISS and Lufthansa among Europe’s worst performers this summer. Swissinfo.ch, 2024.
16. Moubray J. Reliability-Centered Maintenance. Industrial Press, ~2016. Pages: 65-77; 113-123. ISBN: 9780826458775
17. Driels M. R. Probability and Statistics for Engineering Applications. CRC Press, 2013. Pages: 219-236; 284-295. ISBN: 9781439895317
18. Halpern N., Graham A. The Routledge Companion to Air Transport Management. Routledge, 2018. Pages: 88-95; 162-173. ISBN: 9781138672627
19. Bruce P. J., Gao Y., King J. M. C. Airline Operations: A Practical Guide. Routledge, 2017. Pages: 47-53; 109-113. ISBN: 9781138188197
20. Jeppesen. Aviation Maintenance Technician Handbook - General. FAA-H-8083-30A. Jeppesen, 2012. Pages: 231-242; 315-323. ISBN: 9780884875190
21. EASA. Part-145 - Maintenance Organisation Approval. Cologne: EASA, 2023. Pages: 15-27; AMC/GM – 74-80.

22. EASA. Easy Access Rules for Air Operations. Revision 21, Sept. 2023. Cologne: EASA. Pages: CAT.GEN.MPA 100-195; ORO.GEN.110-140.
23. Dombrowski U., Zahn T. Lean Maintenance - State of the Art. Procedia CIRP, 2016. Pages: 326-332. DOI: 10.1016/j.procir.2016.03.204
24. FAA. Order 8900.1 - Flight Standards Information System. Vol. 3, Ch. 30. FAA, 2021. Pages: 1-15; 24-29.
25. FAA. AMT Handbook - Airframe Vol. 1. FAA-H-8083-31A. FAA, 2023. Pages: 217-228; 345-354.
26. Lee H. L. et al. Logistics and Operations Management for Aviation. Springer, 2018. Pages: 52-67; 201-213. ISBN: 9783319772584
27. Nolan M. Fundamentals of Air Traffic Control. 6th Ed. Cengage Learning, 2015. Pages: 289-304; 372-380. ISBN: 9781305956384
28. EASA. Part-M, Part-CAMO. Consolidated version. Cologne: EASA, 2023. Pages: Subparts C, G - 23-37, 51-60.
29. Boeing. Statistical Summary of Commercial Jet Airplane Accidents 1959-2021. Boeing, 2022. Pages: 11-18; 33-39.
30. FAA. System Safety Handbook. FAA, 2000. Pages: 73-82; 145-151.
31. ICAO. Doc 9859 - Safety Management Manual (SMM). 4th Ed. Montreal: ICAO, 2018. Pages: 51-65; 87-91. ISBN: 9789292584391
32. Golaszewski R. Reliability and Maintainability Engineering. John Wiley & Sons, 2020. Pages: 106-114; 231-238. ISBN: 9781118974294
33. ICAO. Doc 9760 - Airworthiness Manual. 3rd Edition. Montreal: ICAO, 2014. Pages: 2-11 – 2-23. ISBN: 9789291947417
34. Balaban E., Saxena A., Goebel K. Model-based Prognostics for Aircraft Systems. Elsevier: Annual Conference of the Prognostics and Health Management Society, 2009. Pages: 1-8; 11-17. DOI: 10.1109/PHM.2009.5416595
35. Airbus. Global Market Forecast 2023-2042. Toulouse: Airbus, 2023. Pages: 67-75; 98-103.
36. ICAO. Annex 8 - Airworthiness of Aircraft. 12th Edition. Montreal: ICAO, 2022. Pages: 4-19; 63-72. ISBN: 9789291950400
37. Kinnison H., Siddiqui T. Aviation Maintenance Management. McGraw-Hill Education, 2013. Pages: 89-102; 123-130. ISBN: 9780071805025
38. Airbus. A320 Maintenance Planning Document (MPD). Rev. 2022. Pages: 6-19; 101-118.
39. FAA. AC 120-124: CAMP. FAA, 2015. Pages: 6-17.
40. Matthes E. Python Crash Course. 2nd Ed. No Starch Press, 2019. Pages: 321-333; 415-421. ISBN: 9781593279288
41. Trivedi K. S. Probability and Statistics with Reliability, Queuing, and Computer Science Applications. 2nd Edition. Hoboken: Wiley-Interscience, 2001. Pages: 23-41; 231-254. ISBN: 9780471333418
42. Wu C.-L. Airline Operations and Delay Management. Routledge, 2016. Pages: 33-41; 97-108. ISBN: 9781472477790

43. Chatterjee S., Zhuang Y., Yin L. Decision Support in Aircraft Maintenance: Applications and Trends. Springer, 2017. Pages: 57-70; 129-138. ISBN: 9783319548530
44. Al-Samarraie H., Zaqout F. Reliability-Based Aircraft Maintenance Optimization. Elsevier, 2018. Pages: 173-182. ISBN: 9780128126683
45. Klein G. Sources of Power: How People Make Decisions. Cambridge: MIT Press, 1999. Pages: 113-126; 201-213. ISBN: 9780262611466
46. Lufthansa Systems GmbH & Co. KG. NetLine/Sched - Product Information. Lufthansa Systems, Jan. 2024, 8 p.
47. Sabre Airline Solutions. Airline Solutions Dictionary - Movement Manager / Movement Control. Sabre Corporation, 2019, 23 p.
48. Jeppesen (A Boeing Company). Operations Control - Commercial Aviation Solutions. Jeppesen Systems, 2022.
49. Medany A., Blumbergs I., Elsakty K. Analysis of International Air Hubs: A Competitiveness Review. Lecture Notes in Intelligent Transportation and Infrastructure, 2023, Part F1379. Pages: 645-654. DOI: 10.1007/978-3-031-25863-3_62.
50. Skuratov K. Directions of Interaction Between Airports and Airlines as Participants in the Aviation Market. 2009. Pages: 27-36.
51. Barrera D. Aircraft Maintenance Programs. Springer, 2022. Pages: 103-118. ISBN 978-3030902629.
52. Price J., Forrest J. Practical Airport Operations, Safety, and Emergency Management: Protocols for Today and the Future. Butterworth-Heinemann, 2016. Pages: 88-101. ISBN 9780128005156.
53. Sales M. Aviation Logistics: The Dynamic Partnership of Air Freight and Supply Chain. Kogan Page Limited, 2016. Pages: 45–52. ISBN 9780749472702.
54. Lu N., Meng B. Risk Analysis of Airplane Upsets in Flight: An Integrated System Framework and Analysis Methodology. Aerospace, 2023, 10(5): 446. Pages: 4-12. DOI: 10.3390/aerospace10050446.
55. Güray Güler M. A Novel Rolling Horizon Based Solution Framework for Scheduling Airplane Maintenance. Süleyman Demirel Üniversitesi Vizyoner Dergisi, 2024, 15(42). Pages: 67-74. DOI: 10.21076/vizyoner.1332082.
56. Chang N. Prediction of Maintenance Hours Based on Maintenance Scenarios. Journal of Physics: Conference Series, 2024, 2764(1): 012086. Pages: 2-6. DOI: 10.1088/1742-6596/2764/1/012086.
57. Karakoc T., Kostić A. Novel Techniques in Maintenance, Repair, and Overhaul. In: Proceedings of the International Symposium on Aviation Technology, MRO, Sares. Springer, 2022. Pages: 201-210. ISBN 3031420403.
58. Interstate Aviation Committee. Aviation Regulations Part 25 - Airworthiness Standards for Transport Category Airplanes. Edition 5 with Amendments, 2024. Pages: 73-85.

59. Vaivads A., Tereščenko J., Šestakovs V. A Model of Interconnection Between Aircraft Equipment Failures and Aircraft 'States' in Flight. Transport and Aerospace Engineering, 2018, 6. Pages: 14-19. DOI: 10.2478/tae-2018-0004.
60. Bogdane R., Vaivads A., Dencic D. Evaluation of Management System Effectiveness in the Preparation of the Aircraft for Flight in Faulty Conditions. Transport and Aerospace Engineering, 2015, Vol. 2, No. 1. Pages: 25-30. DOI: 10.1515/tae-2015-0002.
61. Smirnov N. Manual on Performing Practical Classes in the Discipline: Fundamentals of the Theory of Technical Operation of Aircraft and Motorized Vehicles. Moscow: MGTU GA, 2021. Pages: 39-47.
62. Helmreich R., Klinect J. Models of Threat, Error, and CRM in Flight Operations. In: Proceedings of the Tenth International Symposium on Aviation Psychology. The Ohio State University, 2010. Pages: 345-352.
63. Boeing Commercial Aviation Services. Fleet Maintenance Seminars. Airline Maintenance Program Development, May 4-8, 2015, Seattle, Washington, U.S.A. Pages: 12-19.
64. Rausand M., Barros A., Hoyland A. System Reliability Theory: Models, Statistical Methods, and Applications. 2020. Pages: 211-224. ISBN 9781119373940. DOI: 10.1002/9781119373940.
65. Beck J., McLoughlin B. Maintenance Program Enhancements. Boeing - AeroMagazine, 2006, Qtr 04. Pages: 18-21.
66. A. Zabirov, Z. Zabirov, V. Shestakov. Analysis of Approaches to Assessing Flight Delays Due to Technical Issues at Airline Network Airports Within the Operational Management Framework (2024). DOI: https://doi.org/10.2478/tar-2024-0008;
67. M. Romele, A. Zabirov, R. Chatys, V. Šestakovs. Solving Technological Problems of Flight Services at Airport Involving Composite Materials (2024). Composites Theory and Practice - SCOPUS. DOI: https://doi.org/10.62753/ctp.2024.07.2.2;
68. Goncharenko A. V. Basic Theoretical Provisions of Entropy Approach for Active Systems Communicational Control Assessment in Multi-Alternative Navigational Situations. In: Computer Modeling and Intelligent Systems (CMIS 2023). CEUR Workshop Proceedings, Vol. 3392. CEUR-WS, 2023. Pages: 1-10.
69. IATA. Aircraft Operational Availability. 2nd Edition. International Air Transport Association (IATA), 2022.
70. Ross S. M. *Stochastic Processes*. 2nd ed. Wiley, 1996. Pages: 61-114.
71. Cox D. R. *Renewal Theory*. Methuen, London, 1962. Pages: 35-42.
72. Kleinrock L. Queueing Systems. Volume 1: Theory. Wiley, 1975. Pages: 23-47.
73. Takács L. Introduction to the Theory of Queues. Oxford University Press, 1962. Pages: 89-103.
74. Trivedi K. S. Probability and Statistics with Reliability, Queueing, and Computer Science Applications. Wiley, 2002. Pages: 331-358.
75. Wang W., Pham H. Reliability and Maintenance Engineering. Springer, 2006. Pages: 205-242.

APPENDIX

The appendix includes the full source code of the application tool FlightSync, developed in Python as part of this research. The application implements the proposed model for assessing flight delays due to technical maintenance factors within an airline network. The code is presented below:

```
from datetime import datetime
import os
from tkinter import filedialog
import matplotlib.pyplot as plt
import numpy as np
import tkinter as tk
from tkinter import messagebox
from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg

# === Reference planned time for index calculations (minutes) ===
# Used to compute:
# - Approx. technical regularity index
# - Operational availability (IATA-style)
PLANNED_BLOCK_TIME_MIN = 225.0


def calculate_influence(data, factor_keys, airport_type):
    influences = []
    base_delay = compute_delay(data, airport_type)
    delta_fraction = 0.05  # +5% change of factor

    # If airport type not defined in spares, return zeros
    if airport_type not in data["spares"]:
        return [0] * len(factor_keys)

    for key in factor_keys:
        original_value = get_param(data, key, airport_type)
        if original_value == 0:
            influences.append(0)
            continue
        modified_value = original_value * (1 + delta_fraction)
        set_param(data, key, airport_type, modified_value)
        new_delay = compute_delay(data, airport_type)
        influence = new_delay - base_delay
        influences.append(max(influence, 0))
        set_param(data, key, airport_type, original_value)
    return influences


def get_param(data, key, loc):
    if key == "restoration":
        return data["recovery"][loc]["restoration"]["mean"]
    elif key == "delivery":
```

```
        return data["recovery"][loc]["delivery"]["mean"]
    elif key == "spares":
        return data["spares"][loc]
    elif key == "turnaround":
        return data["turnaround"][loc]
    elif key.startswith("Pij"):
        i, j = map(int, key[3:].split("_"))
        return data["Pij"][i][j]
    elif key == "looping":
        return data["base_ops"]["cycling_prob"]
    elif key == "reserve":
        return data["base_ops"]["reserve_prob"]
    elif key == "wait":
        return data["base_ops"]["wait_mean"]
    return 0


def set_param(data, key, loc, value):
    if key == "restoration":
        data["recovery"][loc]["restoration"]["mean"] = value
    elif key == "delivery":
        data["recovery"][loc]["delivery"]["mean"] = value
    elif key == "spares":
        data["spares"][loc] = value
    elif key == "turnaround":
        data["turnaround"][loc] = value
    elif key.startswith("Pij"):
        i, j = map(int, key[3:].split("_"))
        data["Pij"][i][j] = value
    elif key == "looping":
        data["base_ops"]["cycling_prob"] = value
    elif key == "reserve":
        data["base_ops"]["reserve_prob"] = value
    elif key == "wait":
        data["base_ops"]["wait_mean"] = value


def compute_delay(data, loc):
    # If there is no configuration for this airport type – delay = 0
    if loc not in data["spares"]:
        return 0
    p_sp = data["spares"][loc]
    r = data["recovery"][loc]["restoration"]["mean"]
    d = data["recovery"][loc]["delivery"]["mean"]
    turnaround = data["turnaround"][loc]
    exp_rest = p_sp * r + (1 - p_sp) * (r + d)
    return max(0, exp_rest - turnaround)


def plot_simplified(data):
```

```
    factor_labels = ["Restoration Time", "Supply Chain Access", "Transition Prob.", "Loop & Backup"]
    keys_grouped = ["restoration", "delivery", "Pij0_0", "looping"]
    types = ["T", "TB", "B"]
    colors = ["darkblue", "dodgerblue", "skyblue"]
    results = [calculate_influence(data, keys_grouped, loc) for loc in types]

    x = np.arange(len(factor_labels))
    fig, ax = plt.subplots()
    for r, c, label in zip(results, colors, types):
        ax.plot(x, r, marker="o", label=label, color=c)

    ax.set_xticks(x)
    ax.set_xticklabels(factor_labels)
    ax.set_ylabel("ΔT_DS(ΔYᵢ)")
    ax.set_title("Simplified Polygon of Operational Factor Significance (Grouped)")
    ax.legend()
    fig.tight_layout()
    return fig


def plot_detailed(data):
    keys_detailed = [
        "Pij3_0", "Pij1_0", "Pij2_0", "Pij1_1", "Pij3_1", "Pij3_2",
        "turnaround", "turnaround", "turnaround",
        "restoration", "restoration", "restoration",
        "delivery", "delivery", "delivery",
        "spares", "spares", "spares",
        "looping", "reserve"
    ]
    labels = [
        "P₁₀", "P₂₀", "P₃₀", "P₂₁", "P₃₁", "P₃₂",
        "T_ATᵀ", "T_ATᵀᴮ", "T_ATᴮ",
        "Tᴿᵀ", "Tᴿᵀᴮ", "Tᴿᴮ",
        "Tᴰᵀ", "Tᴰᵀᴮ", "Tᴰᴮ",
        "P_SPᵀ", "P_SPᵀᴮ", "P_SPᴮ",
        "P_LB", "P_RES"
    ]
    types = ["T", "TB", "B"]
    colors = ["darkblue", "dodgerblue", "skyblue"]
    results = [calculate_influence(data, keys_detailed, loc) for loc in types]

    x = np.arange(len(labels))
    fig, ax = plt.subplots()
    for r, c, label in zip(results, colors, types):
        ax.plot(x, r, marker="o", label=label, color=c)

    ax.set_xticks(x)
    ax.set_xticklabels(labels, rotation=45)
    ax.set_ylabel("ΔT_DS(ΔYᵢ)")
```

```
    ax.set_title("Polygon of the Distribution of Operational Factor Significance")
    ax.legend()
    fig.tight_layout()
    return fig


def save_figure(fig):
    path = filedialog.asksaveasfilename(defaultextension=".png", filetypes=[("PNG files",
"*.png")])
    if path:
        fig.savefig(path)


# === Extend results page with plotting buttons ===
def extend_results_page(self):
    def show_plot(fig):
        win = tk.Toplevel(self.root)
        win.title("Graph")
        canvas = FigureCanvasTkAgg(fig, master=win)
        canvas.draw()
        canvas.get_tk_widget().pack(fill="both", expand=True)
        tk.Button(win, text="Save as PNG", command=lambda: save_figure(fig)).pack(pady=5)

    btn_frame = tk.Frame(self.root)
    btn_frame.pack(pady=5)
    tk.Button(btn_frame, text="📈 Plot Simplified Graph",
              command=lambda: show_plot(plot_simplified(self.data))).pack(side="left", padx=5)
    tk.Button(btn_frame, text="📊 Plot Detailed Graph",
              command=lambda: show_plot(plot_detailed(self.data))).pack(side="left", padx=5)


class AircraftMaintenanceGUI:
    def __init__(self, root):
        self.root = root
        self.root.title("FLIGHT SYNC - Aircraft Maintenance Process Model")
        try:
            self.root.iconbitmap("icon.ico")
        except Exception:
            pass
        self.root.geometry("900x700")
        self.root.resizable(False, False)

        self.data = {
            "routes": [],
            "states": {},
            "Pij": [],
            "recovery": {},
            "spares": {},
            "turnaround": {},
            "base_ops": {}
```

```
        }

        self.saved_fields = {}
        self.step_completed = {i: False for i in range(1, 8)}

        self.create_main_menu()

    def clear_screen(self):
        for widget in self.root.winfo_children():
            widget.destroy()

    def create_main_menu(self):
        self.clear_screen()
        frame = tk.Frame(self.root)
        frame.pack(padx=15, pady=15)

        # LOGO
        from tkinter import PhotoImage
        from PIL import Image, ImageTk
        try:
            original_logo = Image.open("logo.png")
            resized_logo = original_logo.resize((160, 160))
            logo_image = ImageTk.PhotoImage(resized_logo)
            logo_label = tk.Label(frame, image=logo_image)
            logo_label.image = logo_image
            logo_label.pack(pady=10)
        except Exception:
            tk.Label(frame, text="FLIGHT SYNC", font=("Segoe UI", 16,
"bold")).pack(pady=10)

        tk.Label(frame, text="FLIGHT SYNC - Aircraft Maintenance Model Interface",
                 font=("Segoe UI", 14, "bold")).pack(pady=10)

        def label(name, step):
            return f"{name} ✓" if self.step_completed[step] else name

        tk.Button(frame, text=label("1. Define Route Network", 1), width=40,
                  command=self.page_routes).pack(pady=3)
        tk.Button(frame, text=label("2. Define Initial Aircraft States", 2), width=40,
                  command=self.page_states).pack(pady=3)
        tk.Button(frame, text=label("3. Define Transition Probabilities", 3), width=40,
                  command=self.page_transitions).pack(pady=3)
        tk.Button(frame, text=label("4. Define Restoration & Delivery Times", 4), width=40,
                  command=self.page_recovery_times).pack(pady=3)
        tk.Button(frame, text=label("5. Define Spares & Turnaround Time", 5), width=40,
                  command=self.page_spares_and_turnaround).pack(pady=3)
        tk.Button(frame, text=label("6. Define Base Operations", 6), width=40,
                  command=self.page_base_operations).pack(pady=3)
        tk.Button(frame, text=label("7. Calculate and Show Results", 7), width=40,
                  command=self.page_calculate_results).pack(pady=3)
```

```python
        tk.Button(frame, text="Exit", width=20, command=self.root.quit).pack(pady=20)

    # === STEP 1: ROUTES ===
    def page_routes(self):
        self.clear_screen()
        self.route_entries = []

        frame = tk.Frame(self.root)
        frame.pack(padx=10, pady=10)

        # Precompute delays placeholder (not used here but kept for structure consistency)
        restoration_metrics = {}
        for loc in ["B", "T", "TB"]:
            if loc in self.data["spares"]:
                p_sp = self.data["spares"][loc]
                r = self.data["recovery"][loc]["restoration"]["mean"]
                d = self.data["recovery"][loc]["delivery"]["mean"]
                turnaround = self.data["turnaround"][loc]
                exp_rest = p_sp * r + (1 - p_sp) * (r + d)
                restoration_metrics[loc] = max(0, exp_rest - turnaround)

        self.saved_fields.setdefault("routes", {"count": "", "entries": []})

        tk.Label(frame, text="STEP 1: DEFINE ROUTE NETWORK",
                 font=("Segoe UI", 12, "bold")).pack(pady=5)
        tk.Label(frame, text="Enter number of route types (L) — UoM: QTY [1–99]").pack()

        self.route_count_entry = tk.Entry(frame)
        self.route_count_entry.pack()
        self.route_count_entry.insert(0, self.saved_fields["routes"]["count"])

        def proceed():
            try:
                L = int(self.route_count_entry.get())
                if not (1 <= L <= 99):
                    raise ValueError("Route count must be 1–99")
            except ValueError as e:
                messagebox.showerror("Input Error", str(e))
                return

            self.saved_fields["routes"]["count"] = str(L)
            self.saved_fields["routes"]["entries"] = self.saved_fields["routes"].get("entries", [])[:L]

            self.clear_screen()
            sub_frame = tk.Frame(self.root)
            sub_frame.pack(padx=10, pady=10)

            for i in range(L):
                row = tk.Frame(sub_frame)
```

```
        row.pack(pady=4)
        tk.Label(row, text=f"Route #{i+1} (e.g. B-T-TB):").pack(side="left")

        entry_route = tk.Entry(row, width=30)
        entry_route.pack(side="left", padx=2)
        entry_route.insert(
            0,
            self.saved_fields["routes"]["entries"][i][0]
            if i < len(self.saved_fields["routes"]["entries"]) else ""
        )

        tk.Label(row, text="Prob. [0.0–1.0]:").pack(side="left")
        entry_prob = tk.Entry(row, width=8)
        entry_prob.pack(side="left")
        entry_prob.insert(
            0,
            self.saved_fields["routes"]["entries"][i][1]
            if i < len(self.saved_fields["routes"]["entries"]) else ""
        )

        self.route_entries.append((entry_route, entry_prob))

    def save():
        routes = []
        try:
            total_prob = 0.0
            for route_entry, prob_entry in self.route_entries:
                route = route_entry.get().strip()
                prob = float(prob_entry.get())
                if not (0.0 <= prob <= 1.0):
                    raise ValueError("Probability must be between 0.0 and 1.0")
                total_prob += prob
                routes.append((route, str(prob)))

            if abs(total_prob - 1.0) > 0.001:
                raise ValueError(
                    f"Sum of all route probabilities must be 1.0 (currently {total_prob:.3f})"
                )
        except Exception as e:
            messagebox.showerror("Input Error", str(e))
            return

        self.saved_fields["routes"]["entries"] = routes
        self.data["routes"] = [(r, float(p)) for r, p in routes]
        self.step_completed[1] = True
        messagebox.showinfo("Saved", "Route data saved successfully.")
        self.create_main_menu()

    tk.Button(sub_frame, text="Submit", command=save).pack(pady=10)
    tk.Button(sub_frame, text="← Back to Menu",
```

```
            command=self.create_main_menu).pack()

    tk.Button(frame, text="Next", command=proceed).pack(pady=10)
    tk.Button(frame, text="← Back to Menu",
            command=self.create_main_menu).pack()
    tk.Label(
        frame,
        text=("Enter the number of route types you want to analyze. This is required to "
            "calculate the average delay across different flight patterns. You will then "
            "be asked to specify the type of each route (e.g., 'B-T-T-TB-B') and its "
            "execution probability. The route type is a sequence of airport categories "
            "along the flight path, and the probability indicates how often this route "
            "is operated. These inputs are essential for calculating the total expected "
            "delay across the route network."),
        fg="gray", wraplength=850, justify="center",
        font=("Segoe UI", 9)
    ).pack(pady=10)

# === STEP 2: INITIAL STATES ===
def page_states(self):
    self.clear_screen()
    self.saved_fields.setdefault("states", {"S0": "", "S1": "", "S2": "", "S3": ""})
    self.state_entries = {}

    frame = tk.Frame(self.root)
    frame.pack(padx=10, pady=10)

    tk.Label(frame, text="STEP 2: DEFINE INITIAL AIRCRAFT STATES",
           font=("Segoe UI", 12, "bold")).pack(pady=5)
    tk.Label(frame, text="Enter probability for each state — total must equal 1.0").pack()
    tk.Label(
        frame,
        text="States: S0 (Ready), S1 (Minor fault), S2 (Major fault), S3 (Unserviceable)"
    ).pack(pady=5)

    for state in ["S0", "S1", "S2", "S3"]:
        row = tk.Frame(frame)
        row.pack(pady=2)
        tk.Label(row, text=f"State {state} [0.0–1.0]:",
               width=25, anchor="w").pack(side="left")
        entry = tk.Entry(row, width=10)
        entry.pack(side="left")
        entry.insert(0, self.saved_fields["states"][state])
        self.state_entries[state] = entry

    def save_states():
        try:
            total = 0.0
            result = {}
            for state, entry in self.state_entries.items():
```

```
                val = float(entry.get())
                if not (0.0 <= val <= 1.0):
                    raise ValueError(f"{state} must be between 0.0 and 1.0")
                total += val
                result[state] = val
            if abs(total - 1.0) > 0.001:
                raise ValueError("Sum of probabilities must equal 1.0")

            self.data["states"] = result
            self.saved_fields["states"] = {k: self.state_entries[k].get()
                                           for k in self.state_entries}
            self.step_completed[2] = True
            messagebox.showinfo("Saved", "Initial aircraft states saved successfully.")
            self.create_main_menu()
        except Exception as e:
            messagebox.showerror("Input Error", str(e))

    tk.Button(frame, text="Submit", command=save_states).pack(pady=10)
    tk.Button(frame, text="← Back to Menu",
              command=self.create_main_menu).pack(pady=5)
    tk.Label(
        frame,
        text=("Enter the probabilities that an aircraft is in each technical state (S0 to S3). "
              "These values represent the initial condition distribution and affect delay
modeling."),
        fg="gray", wraplength=850, justify="center",
        font=("Segoe UI", 9)
    ).pack(pady=10)

  # === STEP 3: TRANSITION PROBABILITIES ===
  def page_transitions(self):
    self.clear_screen()
    frame = tk.Frame(self.root)
    frame.pack(padx=10, pady=10)

    tk.Label(frame, text="STEP 3: TRANSITION PROBABILITIES",
             font=("Segoe UI", 12, "bold")).pack(pady=5)
    tk.Label(frame, text="Enter the transition probabilities between technical states").pack()

    self.saved_fields.setdefault("transitions", {})
    labels = [
        ("P_10", "P_10 (Repair successful – From State S1 to S0)"),
        ("P_20", "P_20 (Repair successful – From State S2 to S0)"),
        ("P_21", "P_21 (Degradation – From State S2 to S1)"),
        ("P_30", "P_30 (Full repair – From State S3 to S0)"),
        ("P_31", "P_31 (Partial repair – From State S3 to S1)"),
        ("P_32", "P_32 (Degraded continuity – From State S3 to S2)")
    ]

    entries = {}
```

```
for key, description in labels:
    row = tk.Frame(frame)
    row.pack(fill="x", pady=2)
    tk.Label(row, text=description, width=55,
             anchor="w").pack(side="left")
    ent = tk.Entry(row)
    ent.pack(side="left")
    ent.insert(0, self.saved_fields.get("transitions", {}).get(key, ""))
    entries[key] = ent

def save_and_continue():
    try:
        pij = [[0.0 for _ in range(3)] for _ in range(4)]
        values = {
            "P_10": float(entries["P_10"].get()),
            "P_20": float(entries["P_20"].get()),
            "P_21": float(entries["P_21"].get()),
            "P_30": float(entries["P_30"].get()),
            "P_31": float(entries["P_31"].get()),
            "P_32": float(entries["P_32"].get())
        }

        # Validate range [0.0 – 1.0]
        for key, val in values.items():
            if not (0.0 <= val <= 1.0):
                raise ValueError(f"{key} must be between 0.0 and 1.0")

        # Assign values to matrix
        pij[1][0] = values["P_10"]
        pij[2][0] = values["P_20"]
        pij[2][1] = values["P_21"]
        pij[3][0] = values["P_30"]
        pij[3][1] = values["P_31"]
        pij[3][2] = values["P_32"]

        # Validate row sums
        if pij[1][0] > 1.0:
            raise ValueError("Sum of transitions from S1 exceeds 1.0")
        if pij[2][0] + pij[2][1] > 1.0:
            raise ValueError("Sum of transitions from S2 exceeds 1.0")
        if pij[3][0] + pij[3][1] + pij[3][2] > 1.0:
            raise ValueError("Sum of transitions from S3 exceeds 1.0")

        self.data["Pij"] = pij
        self.saved_fields["transitions"] = {k: entries[k].get()
                                            for k in entries}
        self.step_completed[3] = True
        messagebox.showinfo("Saved", "Transition probabilities saved.")
        self.create_main_menu()
```

```
            except ValueError as ve:
                messagebox.showerror("Input Error", str(ve))
            except Exception as e:
                messagebox.showerror("Input Error", f"Unexpected error: {e}")

        tk.Button(frame, text="Submit", command=save_and_continue).pack(pady=10)
        tk.Button(frame, text="← Back to Menu",
                  command=self.create_main_menu).pack(pady=10)
        tk.Label(
            frame,
            text=("Enter the transition probabilities between technical states (e.g., from S1 to S0).
"
                  "Values must be in [0.0–1.0]. Each state's outgoing transitions must not exceed
1.0."),
            fg="gray", wraplength=850, justify="center",
            font=("Segoe UI", 9)
        ).pack(pady=10)

    # === STEP 4: RESTORATION & DELIVERY TIMES ===
    def page_recovery_times(self):
        self.clear_screen()
        self.saved_fields.setdefault("recovery", {})
        self.recovery_entries = {}

        frame = tk.Frame(self.root)
        frame.pack(padx=10, pady=10)

        tk.Label(frame, text="STEP 4: RESTORATION & DELIVERY TIMES",
                 font=("Segoe UI", 12, "bold")).pack(pady=5)
        tk.Label(frame, text="Enter average times for restoration and delivery").pack()
        tk.Label(frame, text="UoM: minutes").pack(pady=3)

        for loc in ["B", "T", "TB"]:
            self.saved_fields["recovery"].setdefault(loc, {"rest": "", "deliv": ""})
            row = tk.Frame(frame)
            row.pack(pady=5)
            tk.Label(row, text=f"{loc} Airport — Restoration time:").pack(side="left")
            entry_r = tk.Entry(row, width=8)
            entry_r.insert(0, self.saved_fields["recovery"][loc]["rest"])
            entry_r.pack(side="left", padx=5)
            tk.Label(row, text="Spare parts delivery time:").pack(side="left")
            entry_d = tk.Entry(row, width=8)
            entry_d.insert(0, self.saved_fields["recovery"][loc]["deliv"])
            entry_d.pack(side="left")
            self.recovery_entries[loc] = (entry_r, entry_d)

        def save():
            try:
                result = {}
                for loc in self.recovery_entries:
```

```
            r = float(self.recovery_entries[loc][0].get())
            d = float(self.recovery_entries[loc][1].get())
            if r < 0 or d < 0:
                raise ValueError("Time must be ≥ 0")
            result[loc] = {"restoration": {"mean": r}, "delivery": {"mean": d}}
            self.saved_fields["recovery"][loc]["rest"] = str(r)
            self.saved_fields["recovery"][loc]["deliv"] = str(d)
        self.data["recovery"] = result
        self.step_completed[4] = True
        messagebox.showinfo("Saved", "Recovery and delivery times saved.")
        self.create_main_menu()
    except Exception as e:
        messagebox.showerror("Input Error", str(e))

    tk.Button(frame, text="Submit", command=save).pack(pady=10)
    tk.Button(frame, text="← Back to Menu",
             command=self.create_main_menu).pack()
    tk.Label(
        frame,
        text="Enter probability of spare parts availability and turnaround time.",
        fg="gray", wraplength=850, justify="center",
        font=("Segoe UI", 9)
    ).pack(pady=10)

# === STEP 5: SPARES & TURNAROUND ===
def page_spares_and_turnaround(self):
    self.clear_screen()
    self.saved_fields.setdefault("spares", {})
    self.spares_entries = {}

    frame = tk.Frame(self.root)
    frame.pack(padx=10, pady=10)

    tk.Label(frame, text="STEP 5: SPARES & TURNAROUND TIME",
            font=("Segoe UI", 12, "bold")).pack(pady=5)
    tk.Label(frame, text="Enter probability of spare parts availability and turnaround
time").pack()
    tk.Label(frame, text="UoM: probability [0–1], time [minutes]").pack(pady=3)

    for loc in ["B", "T", "TB"]:
        self.saved_fields["spares"].setdefault(loc, {"prob": "", "turn": ""})
        row = tk.Frame(frame)
        row.pack(pady=5)
        tk.Label(row, text=f"{loc} — Spares Prob.:").pack(side="left")
        entry_p = tk.Entry(row, width=8)
        entry_p.insert(0, self.saved_fields["spares"][loc]["prob"])
        entry_p.pack(side="left", padx=5)
        tk.Label(row, text="Turnaround Time:").pack(side="left")
        entry_t = tk.Entry(row, width=8)
        entry_t.insert(0, self.saved_fields["spares"][loc]["turn"])
```

```
            entry_t.pack(side="left")
            self.spares_entries[loc] = (entry_p, entry_t)

        def save():
            try:
                for loc in self.spares_entries:
                    p = float(self.spares_entries[loc][0].get())
                    t = float(self.spares_entries[loc][1].get())
                    if not (0 <= p <= 1) or t < 0:
                        raise ValueError("Prob. [0–1], Time ≥ 0")
                    self.data["spares"][loc] = p
                    self.data["turnaround"][loc] = t
                    self.saved_fields["spares"][loc]["prob"] = str(p)
                    self.saved_fields["spares"][loc]["turn"] = str(t)
                self.step_completed[5] = True
                messagebox.showinfo("Saved", "Spares and turnaround data saved.")
                self.create_main_menu()
            except Exception as e:
                messagebox.showerror("Input Error", str(e))

        tk.Button(frame, text="Submit", command=save).pack(pady=10)
        tk.Button(frame, text="← Back to Menu",
                  command=self.create_main_menu).pack()
        tk.Label(
            frame,
            text=("Specify the availability of spare parts (0–1) and the turnaround time (in
minutes) — "
                  "i.e., the standard ground time between arrival and next departure at each airport
type."),
            fg="gray", wraplength=850, justify="center",
            font=("Segoe UI", 9)
        ).pack(pady=10)

    # === STEP 6: BASE OPERATIONS ===
    def page_base_operations(self):
        self.clear_screen()
        self.saved_fields.setdefault("base_ops", {"cycling": "", "reserve": "", "wait": ""})
        self.baseop_entries = {}

        frame = tk.Frame(self.root)
        frame.pack(padx=10, pady=10)

        tk.Label(frame, text="STEP 6: BASE OPERATIONS",
                 font=("Segoe UI", 12, "bold")).pack(pady=5)
        tk.Label(frame, text="Enter probabilities and average wait time").pack()
        tk.Label(frame, text="UoM: probability [0–1], time [minutes]").pack(pady=3)

        for label, key in [("Cycle Into Next Flight", "cycling"),
                           ("Reserve Aircraft Available", "reserve"),
                           ("Avg. Wait Time (min)", "wait")]:
```

```
        row = tk.Frame(frame)
        row.pack(pady=4)
        tk.Label(row, text=label + ":").pack(side="left")
        entry = tk.Entry(row, width=10)
        entry.insert(0, self.saved_fields["base_ops"][key])
        entry.pack(side="left")
        self.baseop_entries[key] = entry

    def save():
        try:
            cycling = float(self.baseop_entries["cycling"].get())
            reserve = float(self.baseop_entries["reserve"].get())
            wait = float(self.baseop_entries["wait"].get())
            if not (0 <= cycling <= 1 and 0 <= reserve <= 1 and wait >= 0):
                raise ValueError("Values out of range")
            self.data["base_ops"] = {
                "cycling_prob": cycling,
                "reserve_prob": reserve,
                "wait_mean": wait,
                "wait_var": (0.2 * wait) ** 2
            }
            for k in self.baseop_entries:
                self.saved_fields["base_ops"][k] = self.baseop_entries[k].get()
            self.step_completed[6] = True
            messagebox.showinfo("Saved", "Base operations saved.")
            self.create_main_menu()
        except Exception as e:
            messagebox.showerror("Input Error", str(e))

    tk.Button(frame, text="Submit", command=save).pack(pady=10)
    tk.Button(frame, text="← Back to Menu",
              command=self.create_main_menu).pack()
    tk.Label(
        frame,
        text=("Specify the probability of loop-back occurrence, reserve aircraft availability, "
              "and the average waiting time. These factors reflect the readiness and resilience "
              "of the technical system."),
        fg="gray", wraplength=850, justify="center",
        font=("Segoe UI", 9)
    ).pack(pady=10)

# === STEP 7: RESULTS ===
def page_calculate_results(self):
    self.clear_screen()
    frame = tk.Frame(self.root)
    frame.pack(padx=10, pady=10)

    # Precompute delays for each airport type
    restoration_metrics = {}
    for loc in ["B", "T", "TB"]:
```

```
            if loc in self.data["spares"]:
                p_sp = self.data["spares"][loc]
                r = self.data["recovery"][loc]["restoration"]["mean"]
                d = self.data["recovery"][loc]["delivery"]["mean"]
                turnaround = self.data["turnaround"][loc]
                exp_rest = p_sp * r + (1 - p_sp) * (r + d)
                restoration_metrics[loc] = max(0, exp_rest - turnaround)

        tk.Label(frame, text="STEP 7: RESULTS",
                 font=("Segoe UI", 12, "bold")).pack(pady=5)

        try:
            header_font = ("Segoe UI", 9, "bold")
            normal_font = ("Segoe UI", 9)

            # Delay by airport type
            tk.Label(frame, text="=== Delay by Airport Type ===",
                     font=header_font).pack(anchor="w", padx=10)
            for loc in ["B", "T", "TB"]:
                if loc not in self.data["spares"]:
                    continue
                spare_prob = self.data["spares"][loc]
                rest_mean = self.data["recovery"][loc]["restoration"]["mean"]
                del_mean = self.data["recovery"][loc]["delivery"]["mean"]
                turnaround = self.data["turnaround"][loc]

                expected_restoration_time = spare_prob * rest_mean + (1 - spare_prob) *
(rest_mean + del_mean)
                delay = max(0, expected_restoration_time - turnaround)

                row = tk.Frame(frame)
                row.pack(anchor="w", padx=10)
                tk.Label(row, text=f"{loc} Airport:", font=header_font).pack(side="left")
                tk.Label(row, text=f" Delay: {delay:.2f} min",
                         font=normal_font).pack(side="left")

            tk.Label(frame, text="─" * 80).pack()

            # Delay by routes + network delay
            routes = self.data.get("routes", [])
            total_network_delay = 0.0
            if routes:
                tk.Label(frame, text="=== Delay by Routes ===",
                         font=header_font).pack(anchor="w", padx=10)
                for route_str, prob in routes:
                    segments = route_str.split("-")
                    route_delay = sum(restoration_metrics.get(seg, 0) for seg in segments)
                    weighted = prob * route_delay
                    total_network_delay += weighted
                    line = (f"Route {route_str} (p={prob}): Delay = {route_delay:.2f} min "
```

```
                f"(weighted: {weighted:.2f} min)")
            tk.Label(frame, text=line, anchor="w",
                     font=normal_font, padx=10).pack(fill="x", anchor="w")

        tk.Label(frame, text="─" * 80).pack()

        # Summary indices
        tk.Label(frame, text=f"Total expected technical delay (network):  "
                      f"{total_network_delay:.2f} min",
             font=normal_font).pack(anchor="w", padx=10)

        if PLANNED_BLOCK_TIME_MIN > 0:
            tech_reg_index = max(0.0, 1.0 - total_network_delay /
PLANNED_BLOCK_TIME_MIN)
            op_avail = (PLANNED_BLOCK_TIME_MIN /
                    (PLANNED_BLOCK_TIME_MIN + total_network_delay)) if
total_network_delay >= 0 else 0.0
            tk.Label(frame, text=f"Approx. technical regularity index:   "
                          f"{tech_reg_index:.3f}",
                 font=normal_font).pack(anchor="w", padx=10)
            tk.Label(frame, text=f"Operational availability (IATA-style): {op_avail:.3f}",
                 font=normal_font).pack(anchor="w", padx=10)

        # Base reserve logic
        base = self.data.get("base_ops", {})
        if base.get("cycling_prob", 0) > 0:
            tk.Label(frame, text="=== Base Airport Reserve Logic ===",
                 font=header_font).pack(anchor="w", padx=10)
            reserve_delay = (1 - base.get("reserve_prob", 0)) * base.get("wait_mean", 0)
            rows = [
                ("Probability of Cycling:", base.get("cycling_prob", 0), ""),
                ("Reserve Aircraft Available:", base.get("reserve_prob", 0), ""),
                ("Expected Wait if No Reserve:", base.get("wait_mean", 0), "min"),
                ("Calculated Reserve Delay:", reserve_delay, "min"),
            ]
            for label_text, val, unit in rows:
                row = tk.Frame(frame)
                row.pack(anchor="w", padx=10)
                tk.Label(row, text=label_text,
                     font=header_font).pack(side="left")
                tk.Label(row, text=f" {val:.2f} {unit}",
                     font=normal_font).pack(side="left")

        if all(self.step_completed.get(i) for i in range(1, 7)):
            self.step_completed[7] = True

    except Exception as e:
        tk.Label(frame, text=f"Error: {e}", fg="red").pack()

    tk.Button(frame, text="← Back to Menu",
```

```
                command=self.create_main_menu).pack(pady=10)

        tk.Label(
            frame,
            text=("This step provides the final delay estimates and visual analysis. "
                "Use the buttons below to generate simplified and detailed graphs "
                "showing the influence of each parameter."),
            fg="gray", wraplength=850, justify="center",
            font=("Segoe UI", 9)
        ).pack(pady=10)

        tk.Button(frame, text="💾 Save Results as TXT",
                command=lambda: self.save_results_as_txt(restoration_metrics)).pack(pady=10)

        extend_results_page(self)

    # === SAVE RESULTS TO TXT ===
    def save_results_as_txt(self, restoration_metrics):
        try:
            filepath = filedialog.asksaveasfilename(
                defaultextension=".txt",
                filetypes=[("Text files", "*.txt")]
            )
            if not filepath:
                return

            # Recompute network delay for saving
            total_network_delay = 0.0
            routes = self.data.get("routes", [])
            for route_str, prob in routes:
                segments = route_str.split("-")
                route_delay = sum(restoration_metrics.get(seg, 0) for seg in segments)
                weighted = prob * route_delay
                total_network_delay += weighted

            tech_reg_index = None
            op_avail = None
            if PLANNED_BLOCK_TIME_MIN > 0:
                tech_reg_index = max(0.0, 1.0 - total_network_delay /
PLANNED_BLOCK_TIME_MIN)
                op_avail = (PLANNED_BLOCK_TIME_MIN /
                        (PLANNED_BLOCK_TIME_MIN + total_network_delay)) if
total_network_delay >= 0 else 0.0

            with open(filepath, "w", encoding="utf-8") as f:
                f.write("=== USER INPUT DATA ===\n")
                f.write("Routes:\n")
                for route, prob in self.data.get("routes", []):
                    f.write(f"  {route} (p={prob})\n")
```

```
        f.write("\nInitial States:\n")
        for k, v in self.data.get("states", {}).items():
            f.write(f"  {k}: {v}\n")

        f.write("\nTransition Matrix (Pij):\n")
        for i, row in enumerate(self.data.get("Pij", [])):
            f.write(f"  From S{i}: {row}\n")

        f.write("\nRestoration & Delivery Times:\n")
        for loc, val in self.data.get("recovery", {}).items():
            r = val["restoration"]["mean"]
            d = val["delivery"]["mean"]
            f.write(f"  {loc}: Restoration = {r} min, Delivery = {d} min\n")

        f.write("\nSpares & Turnaround:\n")
        for loc in ["B", "T", "TB"]:
            p = self.data["spares"].get(loc, "-")
            t = self.data["turnaround"].get(loc, "-")
            f.write(f"  {loc}: Spares = {p}, Turnaround = {t} min\n")

        f.write("\nBase Operations:\n")
        base = self.data.get("base_ops", {})
        for key, label in [("cycling_prob", "P_LB"),
                           ("reserve_prob", "P_RES"),
                           ("wait_mean", "T_RES")]:
            f.write(f"  {label}: {base.get(key, '-')}\n")

        f.write("\n\n=== CALCULATED RESULTS ===\n")
        f.write("Delays by Airport Type:\n")
        for loc in ["B", "T", "TB"]:
            delay = restoration_metrics.get(loc, None)
            if delay is not None:
                f.write(f"  {loc}: Delay = {delay:.2f} min\n")

        f.write("\nDelays by Routes:\n")
        for route_str, prob in self.data.get("routes", []):
            segments = route_str.split("-")
            route_delay = sum(restoration_metrics.get(seg, 0) for seg in segments)
            weighted = prob * route_delay
            f.write(
                f"  Route {route_str} (p={prob}): Delay = {route_delay:.2f} min "
                f"(weighted: {weighted:.2f} min)\n"
            )

        f.write(f"\nTotal expected technical delay (network): {total_network_delay:.2f}
min\n")
        if tech_reg_index is not None and op_avail is not None:
            f.write(f"Approx. technical regularity index: {tech_reg_index:.3f}\n")
            f.write(f"Operational availability (IATA-style): {op_avail:.3f}\n")
```

```
            f.write("\nBase Reserve Logic:\n")
            if base.get("cycling_prob", 0) > 0:
                plb = base.get("cycling_prob", 0)
                pres = base.get("reserve_prob", 0)
                tres = base.get("wait_mean", 0)
                reserve_delay = (1 - pres) * tres
                f.write(f"  Probability of Cycling: {plb:.2f} ({plb * 100:.0f}%)\n")
                f.write(f"  Probability of Reserve Aircraft Availability: {pres:.2f} ({pres *
100:.0f}%)\n")
                f.write(f"  Expected Wait if No Reserve: {tres:.2f} min\n")
                f.write(f"  Reserve Delay: {reserve_delay:.2f} min\n")

        messagebox.showinfo("Success", f"Results saved to {os.path.basename(filepath)}")
    except Exception as e:
        messagebox.showerror("Error", str(e))


if __name__ == "__main__":
    root = tk.Tk()
    app = AircraftMaintenanceGUI(root)
    root.mainloop()
```

Table A.1

RAF-AVIA AOG Statistics Data

Nr	A/C Model	A/C Reg	MSN	Date	Defect Occurrence Airport Type	Defect Category	ATA Chapter	Defect Subclass	T_{AT} (min)	Spare Parts Available	T_R (min)	T_D (min)	Reserve Aircraft Used	Loop-back Occurred	Transition Type	Flight Delay Occurred	Actual Tech Delay (min)	T_{REC} (min)	Model Delay max(0, $T_{REC} - T_{AT}$) (min)
1	SF34	YL-RAL	100	2023-03-09	B	S3	29	Hydraulic	60	YES	60	0	YES	NO	S3→S0	NO	0	60	0
2	SF34	YL-RAG	052	2023-03-29	B	S3	34	Avionics/Nav	60	YES	60	0	YES	NO	S3→S0	NO	0	60	0
3	SF34	YL-RAG	052	2023-04-10	TB	S3	32	Landing Gear	45	NO	180	60	YES	NO	S3→S0	YES	195	240	195
4	SF34	YL-RAL	100	2023-04-14	B	S3	32	Landing Gear	60	YES	60	0	YES	NO	S3→S0	NO	0	60	0
5	SF34	YL-RAG	052	2023-04-21	TB	S3	32	Landing Gear	45	YES	180	0	NO	YES	S3→S0	YES	135	180	135
6	SF34	YL-RAL	100	2023-04-28	T	S3	24	Electrical	45	YES	300	0	YES	NO	S3→S0	YES	255	300	255
7	SF34	YL-RAL	100	2023-05-25	B	S3	34	Avionics/Nav	60	YES	60	0	YES	NO	S3→S2	NO	0	60	0
8	SF34	YL-RAL	100	2023-06-24	TB	S3	32	Landing Gear	45	YES	180	0	YES	NO	S3→S0	YES	135	180	135
9	SF34	YL-RAG	052	2023-07-19	TB	S3	27	Flight Controls	45	YES	180	0	NO	NO	S3→S0	YES	135	180	135
10	SF34	YL-RAG	052	2023-08-07	T	S3	21	Pneumatic/AC	45	NO	300	480	YES	NO	S3→S0	YES	735	780	735
11	SF34	YL-RAG	052	2023-08-26	B	S3	28	Fuel	60	NO	60	30	NO	YES	S3→S0	YES	30	90	30
12	SF34	YL-RAG	052	2023-09-09	TB	S3	34	Avionics/Nav	45	NO	180	60	YES	NO	S3→S0	YES	195	240	195
13	SF34	YL-RAG	052	2023-09-12	B	S3	21	Pneumatic/AC	60	YES	60	0	YES	NO	S3→S0	NO	0	60	0
14	SF34	YL-RAL	100	2023-10-24	T	S3	28	Fuel	45	NO	300	480	NO	NO	S3→S0	YES	735	780	735
15	SF34	YL-RAL	100	2023-11-26	TB	S3	21	Pneumatic/AC	45	YES	180	0	NO	NO	S3→S0	YES	135	180	135
16	SF34	YL-RAL	100	2023-12-07	B	S3	29	Hydraulic	60	YES	60	0	NO	NO	S3→S0	NO	0	60	0
17	SF34	YL-RAG	052	2023-12-28	B	S3	32	Landing Gear	60	YES	60	0	NO	YES	S3→S1	NO	0	60	0
18	SF34	YL-RAG	052	2024-01-03	T	S3	73	Engine/Control	45	YES	300	0	YES	NO	S3→S2	YES	255	300	255
19	SF34	YL-RAG	052	2024-02-17	T	S3	28	Fuel	45	NO	300	480	NO	YES	S3→S0	YES	735	780	735
20	SF34	YL-RAG	052	2024-03-03	B	S3	27	Flight Controls	60	YES	60	0	NO	YES	S3→S0	NO	0	60	0
21	SF34	YL-RAG	052	2024-05-28	T	S3	21	Pneumatic/AC	45	NO	300	480	YES	NO	S3→S1	YES	735	780	735
22	SF34	YL-RAG	052	2024-06-11	T	S3	32	Landing Gear	45	NO	300	480	NO	YES	S3→S0	YES	735	780	735
23	SF34	YL-RAG	052	2024-06-12	B	S3	32	Landing Gear	60	YES	60	0	NO	YES	S3→S2	NO	0	60	0
24	SF34	YL-RAG	052	2024-07-06	B	S3	29	Hydraulic	60	YES	60	0	NO	NO	S3→S2	NO	0	60	0
25	SF34	YL-RAG	052	2024-08-13	B	S3	34	Avionics/Nav	60	YES	60	0	NO	NO	S3→S0	NO	0	60	0
26	SF34	YL-RAG	052	2024-08-16	B	S3	21	Pneumatic/AC	60	YES	60	0	NO	YES	S3→S0	NO	0	60	0
27	SF34	YL-RAL	100	2024-09-15	T	S3	28	Fuel	45	YES	300	0	YES	NO	S3→S0	YES	255	300	255

Nr	A/C Model	A/C Reg	MSN	Date	Defect Occurrence Airport Type	Defect Category	ATA Chapter	Defect Subclass	T_{AT} (min)	Spare Parts Available	T_R (min)	T_D (min)	Reserve Aircraft Used	Loop-back Occurred	Transition Type	Flight Delay Occurred	Actual Tech Delay (min)	T_{REC} (min)	Model Delay max(0, $T_{REC} - T_{AT}$) (min)
28	SF34	YL-RAL	100	2024-09-30	T	S3	29	Hydraulic	45	NO	300	480	NO	NO	S3→S0	YES	735	780	735
29	SF34	YL-RAG	052	2024-11-06	TB	S3	24	Electrical	45	YES	180	0	NO	NO	S3→S1	YES	135	180	135
30	SF34	YL-RAL	100	2024-11-11	TB	S3	29	Hydraulic	45	YES	180	0	NO	NO	S3→S0	YES	135	180	135
31	SF34	YL-RAL	100	2024-11-29	B	S3	24	Electrical	60	YES	60	0	NO	YES	S3→S0	NO	0	60	0
32	SF34	YL-RAL	100	2025-02-27	TB	S3	32	Landing Gear	45	NO	180	60	NO	NO	S3→S1	YES	195	240	195
33	SF34	YL-RAL	100	2025-04-17	T	S3	32	Landing Gear	45	NO	300	480	NO	NO	S3→S1	YES	735	780	735
34	SF34	YL-RAL	100	2025-05-07	B	S3	32	Landing Gear	60	YES	60	0	NO	NO	S3→S2	NO	0	60	0
35	SF34	YL-RAG	052	2025-05-22	B	S3	28	Fuel	60	YES	60	0	NO	NO	S3→S0	NO	0	60	0
36	SF34	YL-RAG	052	2025-05-31	TB	S3	73	Engine/Control	45	YES	180	0	YES	NO	S3→S1	YES	135	180	135
37	SF34	YL-RAG	052	2025-07-06	B	S3	24	Electrical	60	YES	60	0	NO	NO	S3→S0	NO	0	60	0
38	SF34	YL-RAL	100	2025-09-25	B	S3	21	Pneumatic/AC	60	YES	60	0	NO	YES	S3→S0	NO	0	60	0
39	SF34	YL-RAG	052	2025-10-24	B	S3	29	Hydraulic	60	YES	60	0	NO	YES	S3→S2	NO	0	60	0
40	SF34	YL-RAG	052	2025-11-25	B	S3	34	Avionics/Nav	60	YES	60	0	NO	YES	S3→S0	NO	0	60	0

Anvar Zabirov was born in 1991 in Tashkent, Uzbekistan. He obtained his Bachelor's degree (2016) and a Master's degree (2018) in Aviation Transport from Riga Technical University. Since 2017, he has been professionally active in the aviation industry, working in the field of aircraft technical operations and maintenance management.

His academic and professional work focuses on aircraft technical operations, maintenance management, and the improvement of flight regularity. His research interests include optimization of aircraft technical operation parameters, aircraft technical reliability, delay analysis, and the development of operational decision-support tools for airline maintenance management.